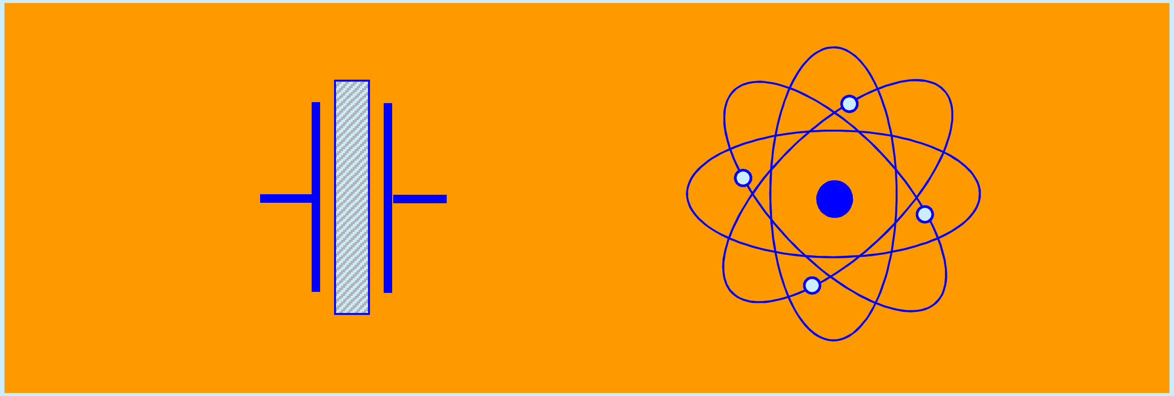
Rev. 8.5.3.9

Quartz Crystal Resonators and Oscillators

For Frequency Control and Timing Applications - A Tutorial

November 2008



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Electronics Applications of Quartz Crystals

Military & Aerospace
- Communications
- Navigation
- IFF
- Radar
- Sensors
- Guidance systems
- Fuzes
- Electronic warfare
- Sonobouys

Research & Metrology
- Atomic clocks
- Instruments
- Astronomy & geodesy
- Space tracking
- Celestial navigation

Industrial
- Communications
- Telecommunications
- Mobile/cellular/portable radio, telephone & pager
- Aviation
- Marine
- Navigation
- Instrumentation
- Computers
- Digital systems
- CRT displays
- Disk drives
- Modems
- Tagging/identification
- Utilities
- Sensors

Consumer
- Watches & clocks
- Cellular & cordless phones, pagers
- Radio & hi-fi equipment
- TV & cable TV
- Personal computers
- Digital cameras
- Video camera/recorder
- CB & amateur radio
- Toys & games
- Pacemakers
- Other medical devices
- Other digital devices

Automotive

Engine control, stereo, clock, yaw stability control, trip computer, GPS

Frequency Control Device Market

(estimates, as of ~2006)

Technology	Units per year	Unit price, typical	Worldwide market, $/year
Quartz Crystal Resonators & Oscillators	~ 3 x 10^9	~$1 ($0.1 to 3,000)	~$4B
Atomic Frequency Standards (see chapter 6)			
Hydrogen maser	~ 20	$100,000	$2M
Cesium beam frequency standard	~ 500	$50,000	$25M
Rubidium cell frequency standard	~ 50,000	$2,000	$100M

Navigation

Precise time is essential to precise navigation. Historically, navigation has been a principal motivator in man's search for better clocks. Even in ancient times, one could measure latitude by observing the stars' positions. However, to determine longitude, the problem became one of timing. Since the earth makes one revolution in 24 hours, one can determine longitude form the time difference between local time (which was determined from the sun's position) and the time at the Greenwich meridian (which was determined by a clock):

Longitude in degrees = (360 degrees/24 hours) x t in hours.

In 1714, the British government offered a reward of 20,000 pounds to the first person to produce a clock that allowed the determination of a ship's longitude to 30 nautical miles at the end of a six week voyage (i.e., a clock accuracy of three seconds per day). The Englishman John Harrison won the competition in 1735 for his chronometer invention.

Today's electronic navigation systems still require ever greater accuracies. As electromagnetic waves travel 300 meters per microsecond, e.g., if a vessel's timing was in error by one millisecond, a navigational error of 300 kilometers would result. In the Global Positioning System (GPS), atomic clocks in the satellites and quartz oscillators in the receivers provide nanosecond-level accuracies. The resulting (worldwide) navigational accuracies are about ten meters (see chapter 8 for further details about GPS).

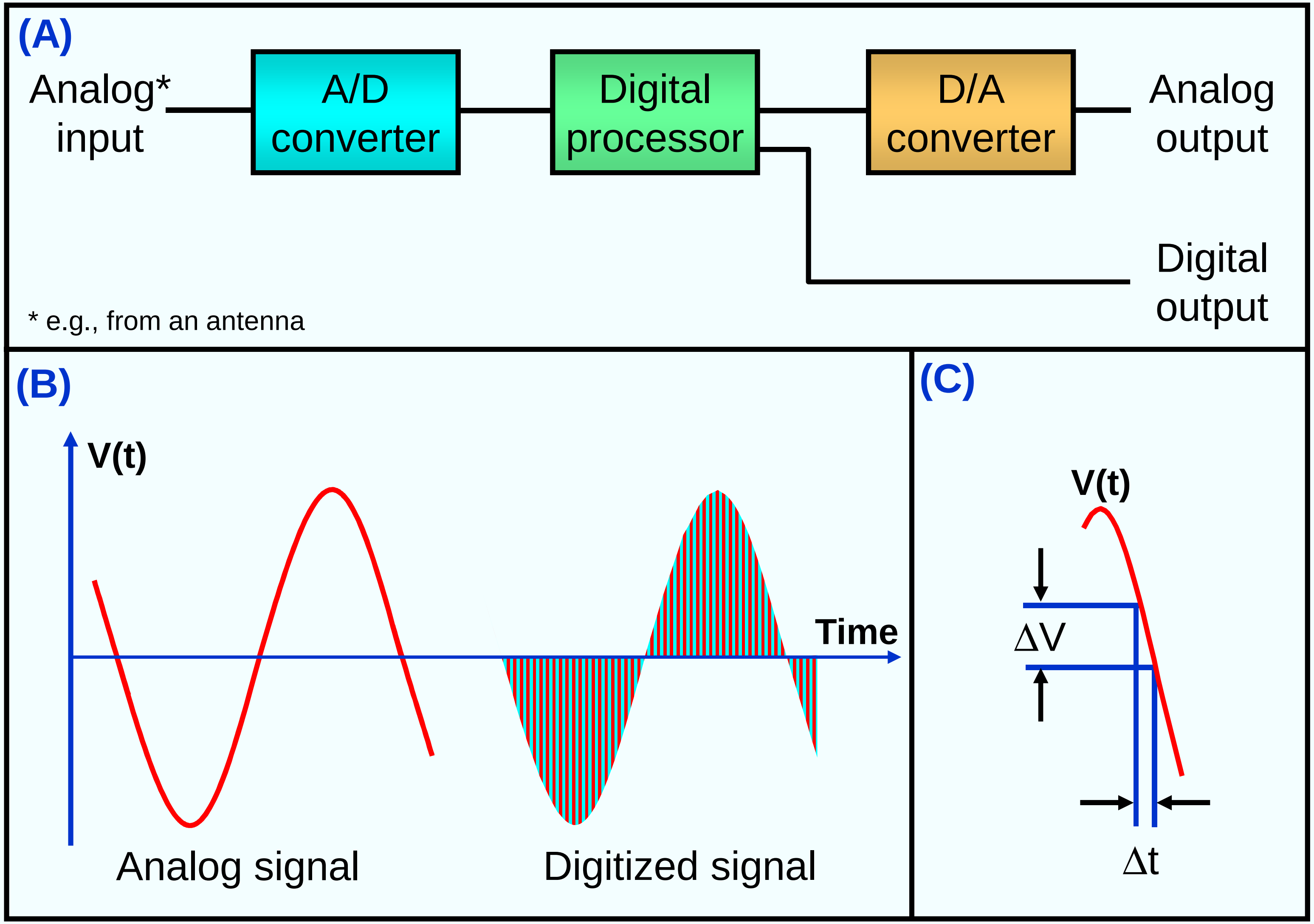
Commercial Two-way Radio

Historically, as the number of users of commercial two-way radios have grown, channel spacings have been narrowed, and higher-frequency spectra have had to be allocated to accommodate the demand. Narrower channel spacings and higher operating frequencies necessitate tighter frequency tolerances for both the transmitters and the receivers. In 1940, when only a few thousand commercial broadcast transmitters were in use, a 500 ppm tolerance was adequate. Today, the oscillators in the many millions of cellular telephones (which operate at frequency bands above 800 MHz) must maintain a frequency tolerance of 2.5 ppm and better. The 896-901 MHz and 935-940 MHz mobile radio bands require frequency tolerances of 0.1 ppm at the base station and 1.5 ppm at the mobile station.

The need to accommodate more users will continue to require higher and higher frequency accuracies. For example, a NASA concept for a personal satellite communication system would use walkie-talkie-like hand-held terminals, a 30 GHz uplink, a 20 GHz downlink, and a 10 kHz channel spacing. The terminals' frequency accuracy requirement is a few parts in 10^8.

Digital Processing of Analog Signals

The Effect of Timing Jitter



Digital Network Synchronization

- Synchronization plays a critical role in digital telecommunication systems. It ensures that information transfer is performed with minimal buffer overflow or underflow events, i.e., with an acceptable level of "slips." Slips cause problems, e.g., missing lines in FAX transmission, clicks in voice transmission, loss of encryption key in secure voice transmission, and data retransmission.

- In AT&T's network, for example, timing is distributed down a hierarchy of nodes. A timing source-receiver relationship is established between pairs of nodes containing clocks. The clocks are of four types, in four "stratum levels."

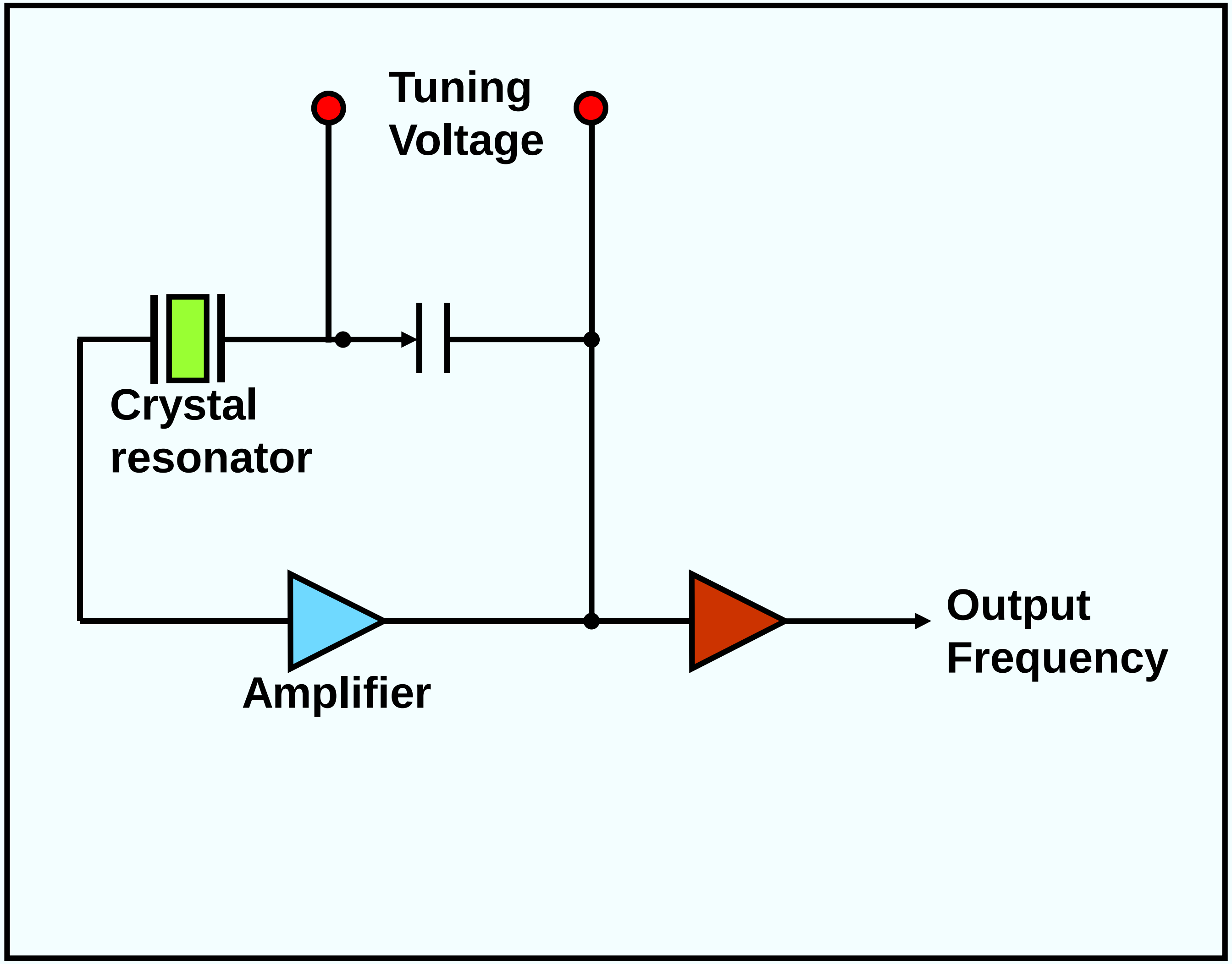
Stratum	Accuracy (Free Running) Long Term	Accuracy (Free Running) Per 1st Day	Clock Type	Number Used
1	1×10^{-11}	N.A.	GPS W/Two Rb	16
2	1.6×10^{-8}	1×10^{-10}	Rb Or OCXO	~200
3	4.6×10^{-6}	3.7×10^{-7}	OCXO Or TCXO	1000's
4	3.2×10^{-5}	N.A.	XO	~1 million

Phase Noise in PLL and PSK Systems

The phase noise of oscillators can lead to erroneous detection of phase transitions, i.e., to bit errors, when phase shift keyed (PSK) digital modulation is used. In digital communications, for example, where 8-phase PSK is used, the maximum phase tolerance is ±22.5°, of which ±7.5° is the typical allowable carrier noise contribution. Due to the statistical nature of phase deviations, if the RMS phase deviation is 1.5°, for example, the probability of exceeding the ±7.5° phase deviation is 6×10^{-7}, which can result in a bit error rate that is significant in some applications.

Shock and vibration can produce large phase deviations even in "low noise" oscillators. Moreover, when the frequency of an oscillator is multiplied by N, the phase deviations are also multiplied by N. For example, a phase deviation of 10^{-3} radian at 10 MHz becomes 1 radian at 10 GHz. Such large phase excursions can be catastrophic to the performance of systems, e.g., of those which rely on phase locked loops (PLL) or phase shift keying (PSK). Low noise, acceleration insensitive oscillators are essential in such applications.

Crystal Oscillator



Oscillation

- At the frequency of oscillation, the closed loop phase shift $= 2n\pi$.
- When initially energized, the only signal in the circuit is noise. That component of noise, the frequency of which satisfies the phase condition for oscillation, is propagated around the loop with increasing amplitude. The rate of increase depends on the excess; i.e., small-signal, loop gain and on the BW of the crystal in the network.
- The amplitude continues to increase until the amplifier gain is reduced either by nonlinearities of the active elements ("self limiting") or by some automatic level control.
- At steady state, the closed-loop gain = 1.

Oscillation and Stability

- If a phase perturbation $\Delta\phi$ occurs, the frequency must shift Δf to maintain the $2n\pi$ phase condition, where $\Delta f/f = -\Delta\phi/2Q_L$ for a series-resonance oscillator, and Q_L is loaded Q of the crystal in the network. The "phase slope," $d\phi/df$ is proportional to Q_L in the vicinity of the series resonance frequency (also see "Equivalent Circuit" and "Frequency vs. Reactance" in Chapt. 3).

- Most oscillators operate at "parallel resonance," where the reactance vs. frequency slope, dX/df, i.e., the "stiffness," is inversely proportional to C_1, the motional capacitance of the crystal unit.

- For maximum frequency stability with respect to phase (or reactance) perturbations in the oscillator loop, the phase slope (or reactance slope) must be maximum, i.e., C_1 should be minimum and Q_L should be maximum. A quartz crystal unit's high Q and high stiffness makes it the primary frequency (and frequency stability) determining element in oscillators.

Tunability and Stability

Making an oscillator tunable over a wide frequency range degrades its stability because making an oscillator susceptible to intentional tuning also makes it susceptible to factors that result in unintentional tuning. The wider the tuning range, the more difficult it is to maintain a high stability. For example, if an OCXO is designed to have a short term stability of 1×10^{-12} for some averaging time and a tunability of 1×10^{-7}, then the crystal's load reactance must be stable to 1×10^{-5} for that averaging time. Achieving such stability is difficult because the load reactance is affected by stray capacitances and inductances, by the stability of the varactor's capacitance vs. voltage characteristic, and by the stability of the voltage on the varactor. Moreover, the 1×10^{-5} load reactance stability must be maintained not only under benign conditions, but also under changing environmental conditions (temperature, vibration, radiation, etc.).

Whereas a high stability, ovenized 10 MHz voltage controlled oscillator may have a frequency adjustment range of 5×10^{-7} and an aging rate of 2×10^{-8} per year, a wide tuning range 10 MHz VCXO may have a tuning range of 50 ppm and an aging rate of 2 ppm per year.

Oscillator Acronyms

Most Commonly Used:

- **XO…………..Crystal Oscillator**
- **VCXO………Voltage Controlled Crystal Oscillator**
- **OCXO………Oven Controlled Crystal Oscillator**
- **TCXO………Temperature Compensated Crystal Oscillator**

Others:

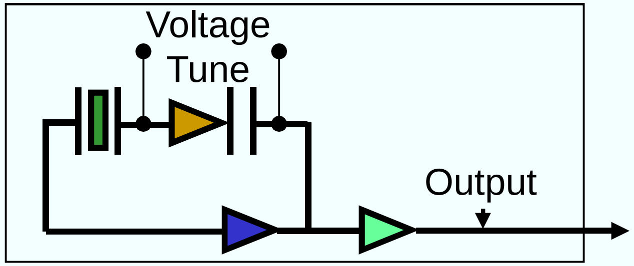
- **TCVCXO..…Temperature Compensated/Voltage Controlled Crystal Oscillator**
- **OCVCXO.….Oven Controlled/Voltage Controlled Crystal Oscillator**
- **MCXO………Microcomputer Compensated Crystal Oscillator**
- **RbXO……….Rubidium-Crystal Oscillator**

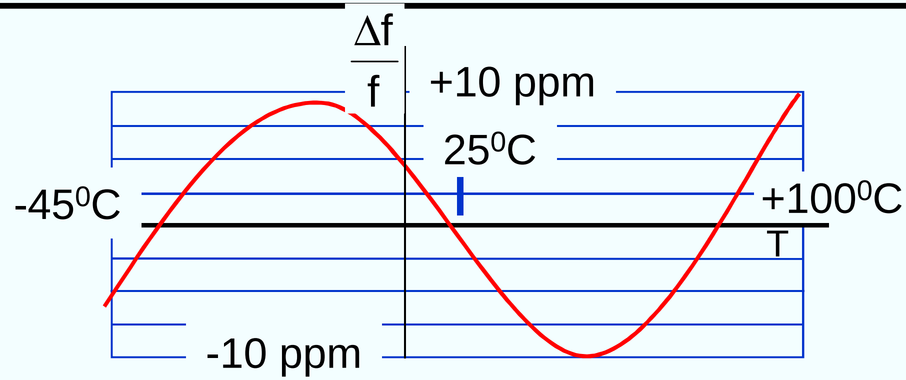
Crystal Oscillator Categories

The three categories, based on the method of dealing with the crystal unit's frequency vs. temperature (f vs. T) characteristic, are:

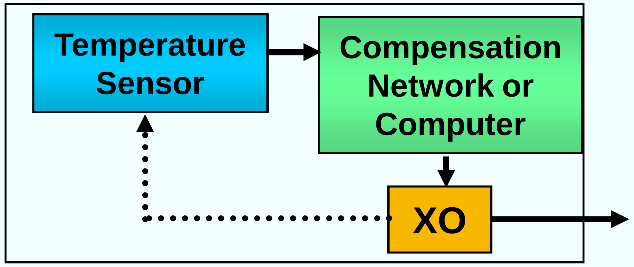
- **XO, crystal oscillator,** does not contain means for reducing the crystal's f vs. T characteristic (also called PXO-packaged crystal oscillator).
- **TCXO, temperature compensated crystal oscillator,** in which, e.g., the output signal from a temperature sensor (e.g., a thermistor) is used to generate a correction voltage that is applied to a variable reactance (e.g., a varactor) in the crystal network. The reactance variations compensate for the crystal's f vs. T characteristic. Analog TCXO's can provide about a 20X improvement over the crystal's f vs. T variation.
- **OCXO, oven controlled crystal oscillator,** in which the crystal and other temperature sensitive components are in a stable oven which is adjusted to the temperature where the crystal's f vs. T has zero slope. OCXO's can provide a >1000X improvement over the crystal's f vs. T variation.

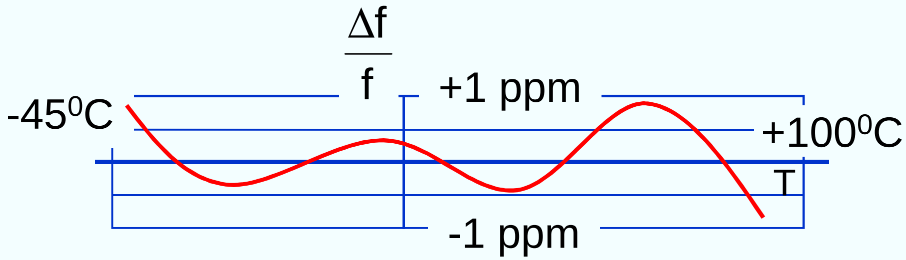
Crystal Oscillator Categories



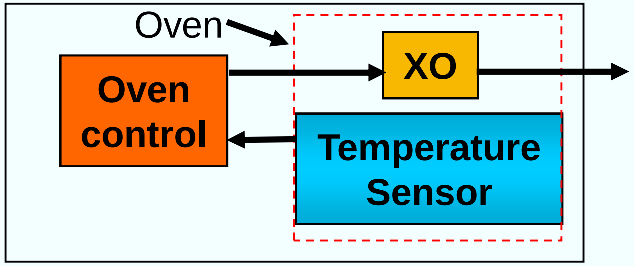


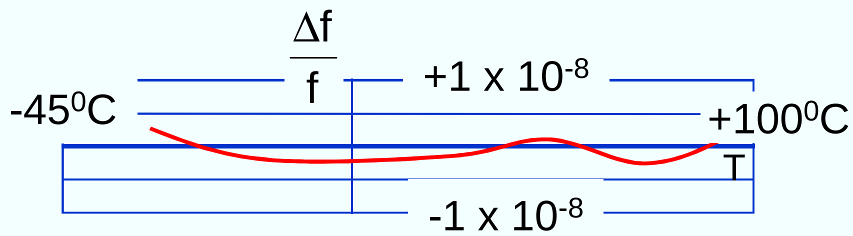
- Crystal Oscillator (XO)





- Temperature Compensated (TCXO)





- Oven Controlled (OCXO)

Hierarchy of Oscillators

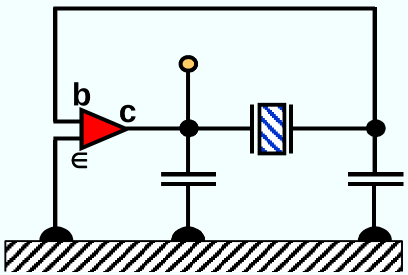
Oscillator Type*	Accuracy**	Typical Applications
• Crystal oscillator (XO)	10^{-5} to 10^{-4}	Computer timing
• Temperature compensated crystal oscillator (TCXO)	10^{-6}	Frequency control in tactical radios
• Microcomputer compensated crystal oscillator (MCXO)	10^{-8} to 10^{-7}	Spread spectrum system clock
• Oven controlled crystal oscillator (OCXO)	10^{-8} (with 10^{-10} per g option)	Navigation system clock & frequency standard, MTI radar
• Small atomic frequency standard (Rb, RbXO)	10^{-9}	C^3 satellite terminals, bistatic, & multistatic radar
• High performance atomic standard (Cs)	10^{-12} to 10^{-11}	Strategic C^3, EW

* Sizes range from <1cm^3 for clock oscillators to > 30 liters for Cs standards
Costs range from <$1 for clock oscillators to > $50,000 for Cs standards.

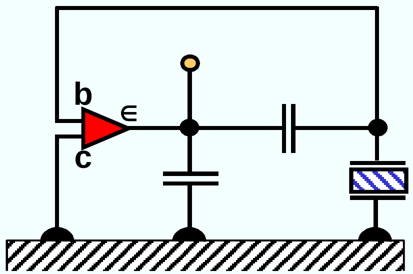
** Including environmental effects (e.g., -40^{o}C to +75^{o}C) and one year of aging.

Oscillator Circuit Types

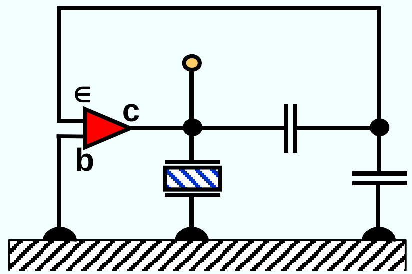
Of the numerous oscillator circuit types, three of the more common ones, the Pierce, the Colpitts and the Clapp, consist of the same circuit except that the rf ground points are at different locations. The Butler and modified Butler are also similar to each other; in each, the emitter current is the crystal current. The gate oscillator is a Pierce-type that uses a logic gate plus a resistor in place of the transistor in the Pierce oscillator. (Some gate oscillators use more than one gate).



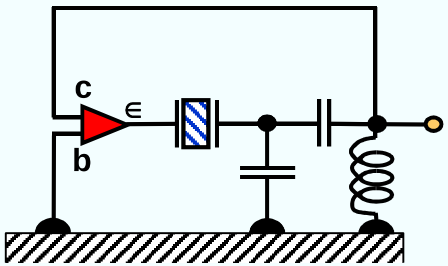
Pierce



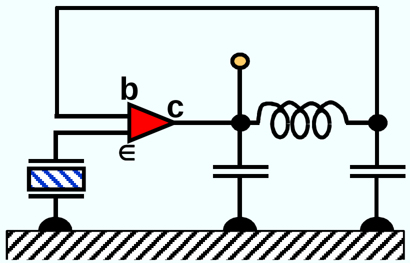
Colpitts



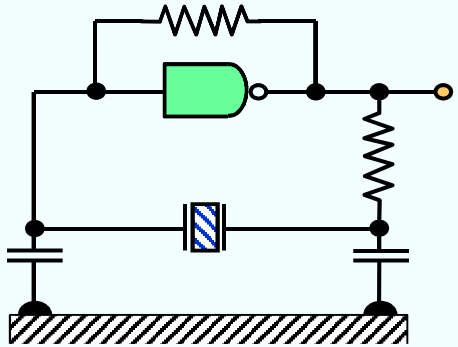
Clapp



Butler

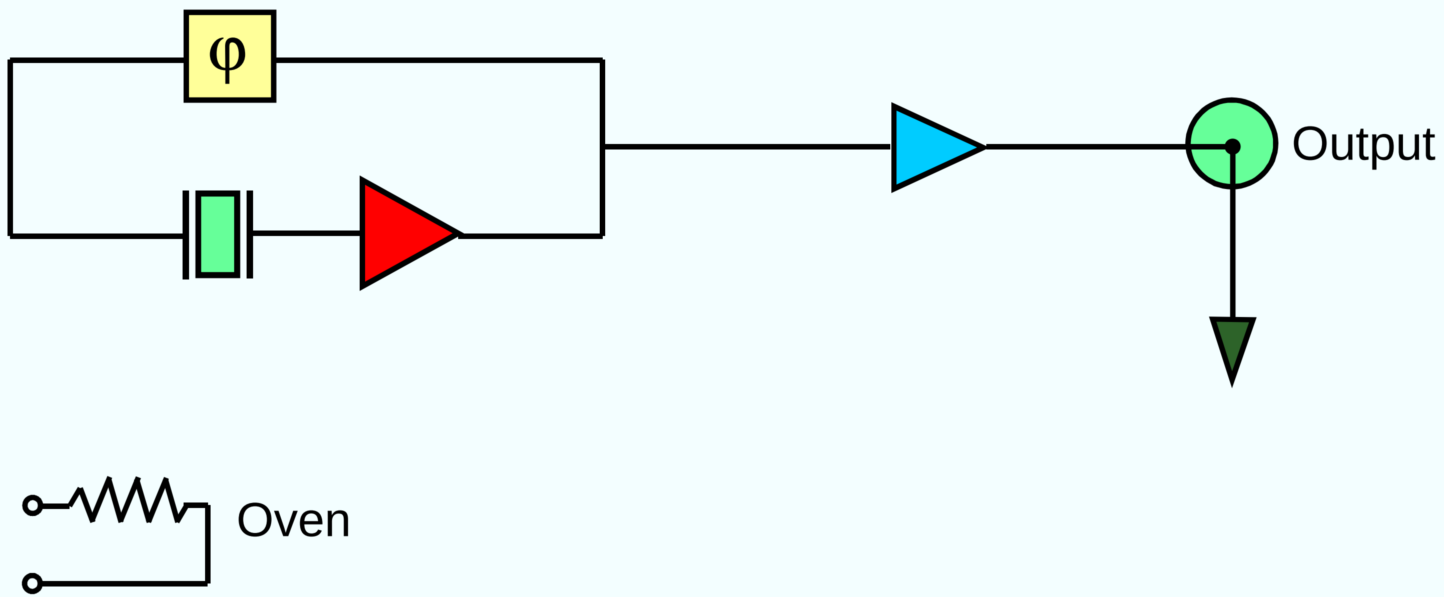


Modified Butler



Gate

OCXO Block Diagram



Each of the three main parts of an OCXO, i.e., the crystal, the sustaining circuit, and the oven, contribute to instabilities. The various instabilities are discussed in the rest of chapter 3 and in chapter 4.

Oscillator Instabilities - General Expression

$$\frac{\Delta f}{f_{oscillator}} \approx \frac{\Delta f}{f_{resonator}} + \frac{1}{2Q_L}\left[1+\left(\frac{2f_f Q_L}{f}\right)^2\right]^{-1/2} d\varphi(f_f)$$

where Q_L = loaded Q of the resonator, and $d\phi(f_f)$ is a small change in loop phase at offset frequency f_f away from carrier frequency f. Systematic phase changes and phase noise within the loop can originate in either the resonator or the sustaining circuits. Maximizing Q_L helps to reduce the effects of noise and environmentally induced changes in the sustaining electronics. In a properly designed oscillator, the short-term instabilities are determined by the resonator at offset frequencies smaller than the resonator's half-bandwidth, and by the sustaining circuit and the amount of power delivered from the loop for larger offsets.

Instabilities due to Sustaining Circuit

• **Load reactance change** - adding a load capacitance to a crystal changes the frequency by

$$\delta f \equiv \frac{\Delta f}{f} \cong \frac{C_1}{2(C_0 + C_L)}$$

$$\text{then,} \quad \frac{\Delta(\delta f)}{\Delta C_L} \cong -\frac{C_1}{2(C_0 + C_L)^2}$$

• **Example**: If C_0 = 5 pF, C_1 = 14fF and C_L = 20pF, then a ΔC_L = 10 fF (= 5 X 10^{-4}) causes ≈1 X 10^{-7} frequency change, and a C_L aging of 10 ppm per day causes 2 X 10^{-9} per day of oscillator aging.

• **Drive level changes:** Typically 10^{-8} per ma^2 for a 10 MHz 3rd SC-cut.

• **DC bias** on the crystal also contributes to oscillator aging.

Oscillator Instabilities - Tuned Circuits

Many oscillators contain tuned circuits - to suppress unwanted modes, as matching circuits, and as filters. The effects of small changes in the tuned circuit's inductance and capacitance is given by:

$$\frac{\Delta f}{f}_{\text{oscillator}} \approx \frac{d\varphi(f_f)}{2Q_L} \approx \left(\frac{1}{1+\frac{2f_f}{BW}}\right)\left(\frac{Q_c}{Q}\right)\left(\frac{dC_c}{C_c}+\frac{dL_c}{L_c}\right)$$

where BW is the bandwidth of the filter, f_f is the frequency offset of the center frequency of the filter from the carrier frequency, Q_L is the loaded Q of the resonator, and Q_c, L_c and C_c are the tuned circuit's Q, inductance and capacitance, respectively.

Oscillator Instabilities - Circuit Noise

Flicker PM noise in the sustaining circuit causes flicker FM contribution to the oscillator output frequency given by:

$$\mathcal{L}_{osc}\left(f_f\right) = \mathcal{L}_{ckt}\left(1\text{Hz}\right)\frac{f^2}{4f_f^3 Q_L^2}$$

and

$$\sigma_y\left(\tau\right) = \frac{1}{Q_L}\sqrt{\ln 2\,\mathcal{L}_{ckt}\left(1\text{Hz}\right)}$$

where f_f is the frequency offset from the carrier frequency f, Q_L is the loaded Q of the resonator in the circuit, $\mathcal{L}_{ckt}$ (1Hz) is the flicker PM noise at f_f = 1Hz, and τ is any measurement time in the flicker floor range. For $Q_L = 10^6$ and $\mathcal{L}_{ckt}$ (1Hz) = -140dBc/Hz, $\sigma_y(\tau) = 8.3 \times 10^{-14}$. ($\mathcal{L}_{ckt}$ (1Hz) = -155dBc/Hz has been achieved.)

Oscillator Instabilities - External Load

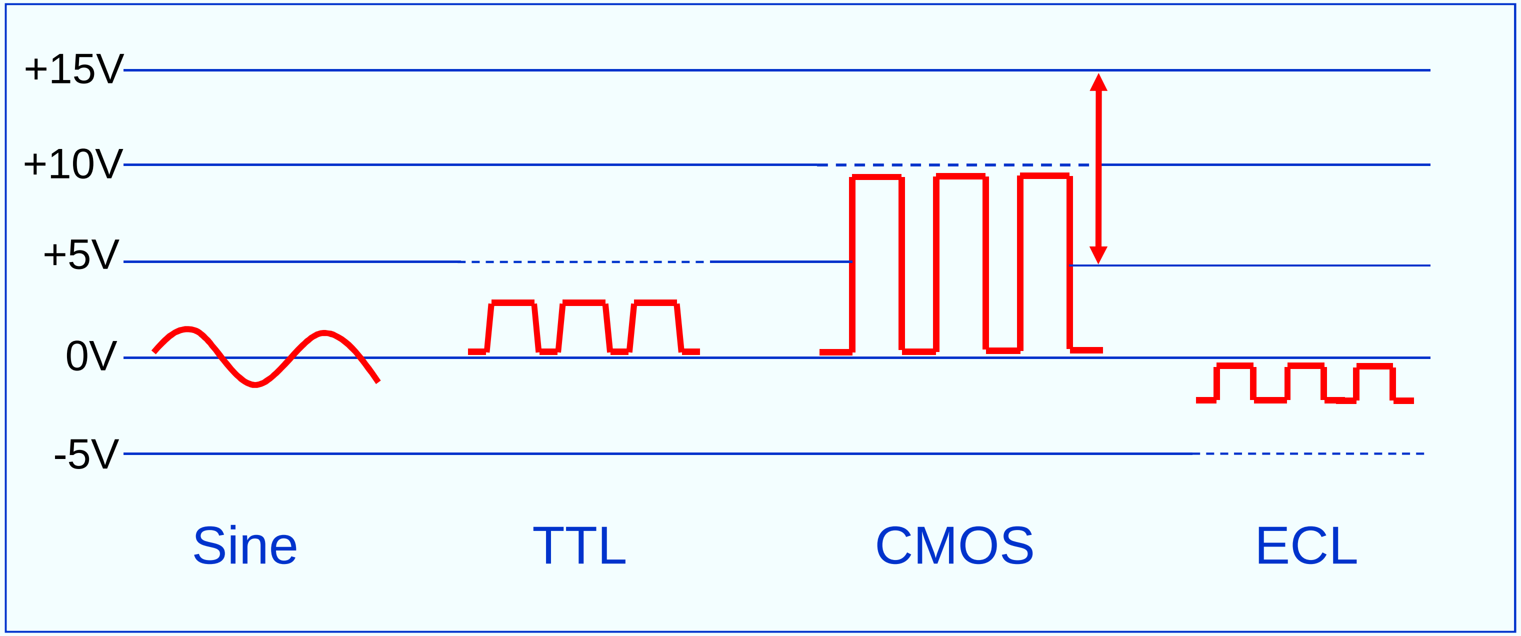
If the external load changes, there is a change in the amplitude or phase of the signal reflected back into the oscillator. The portion of that signal which reaches the oscillating loop changes the oscillation phase, and hence the frequency by

$$\frac{\Delta f}{f_{\text{oscillator}}} \approx \frac{d\phi(f_f)}{2Q} \approx \left(\frac{1}{2Q}\right)\left(\frac{\Gamma - 1}{\Gamma + 1}\right)(\sin\theta)\sqrt{\text{isolation}}$$

where Γ is the VSWR of the load, and θ is the phase angle of the reflected wave; e.g., if Q ~ 10^{6}, and isolation ~40 dB (i.e., ~10^{-4}), then the worst case (100% reflection) pulling is ~5 x 10^{-9}. A VSWR of 2 reduces the maximum pulling by only a factor of 3. The problem of load pulling becomes worse at higher frequencies, because both the Q and the isolation are lower.

Oscillator Outputs

Most users require a sine wave, a TTL-compatible, a CMOS-compatible, or an ECL-compatible output. The latter three can be simply generated from a sine wave. The four output types are illustrated below, with the dashed lines representing the supply voltage inputs, and the bold solid lines, the outputs. (There is no "standard" input voltage for sine wave oscillators. The input voltages for CMOS typically range from 1V to 10V.)

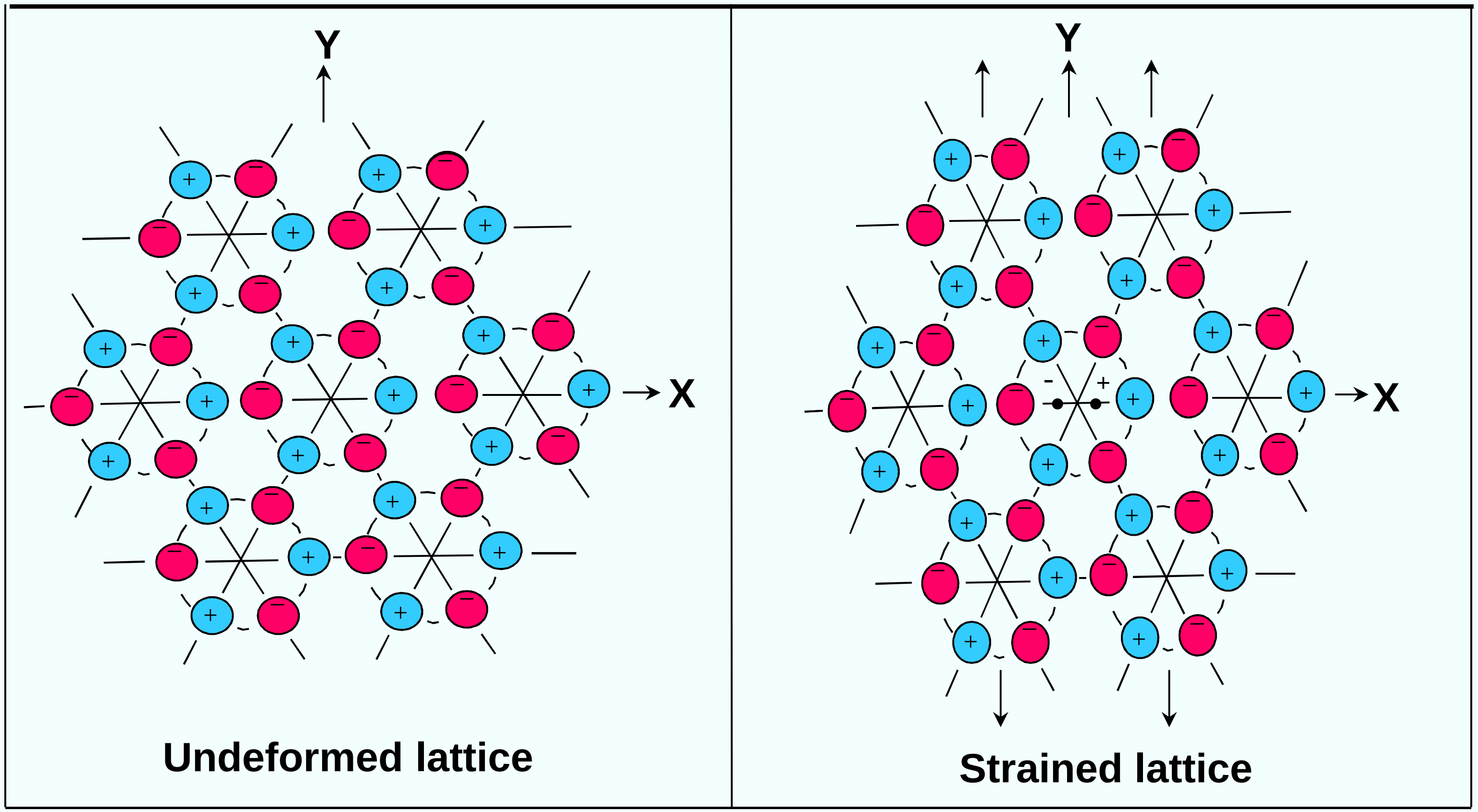


Why Quartz?

Quartz is the only material known that possesses the following combination of properties:

- Piezoelectric ("pressure-electric"; piezein = to press, in Greek)
- Zero temperature coefficient cuts exist
- Stress compensated cut exists
- Low loss (i.e., high Q)
- Easy to process; low solubility in everything, under "normal" conditions, except the fluoride and hot alkali etchants; hard but not brittle
- Abundant in nature; easy to grow in large quantities, at low cost, and with relatively high purity and perfection. Of the man-grown single crystals, quartz, at ~3,000 tons per year, is second only to silicon in quantity grown (3 to 4 times as much Si is grown annually, as of 1997).

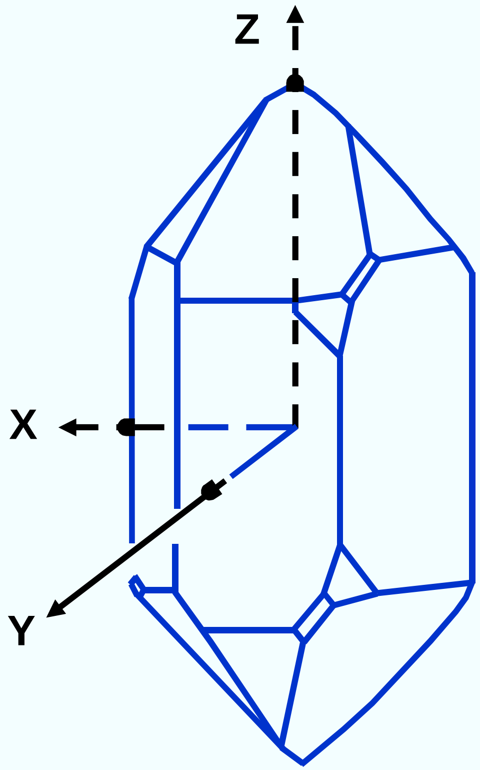
The Piezoelectric Effect



The piezoelectric effect provides a coupling between the mechanical properties of a piezoelectric crystal and an electrical circuit.

The Piezoelectric Effect in Quartz

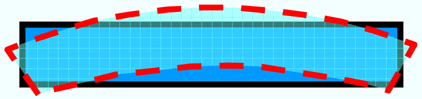
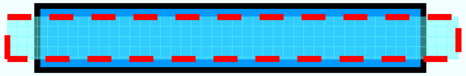
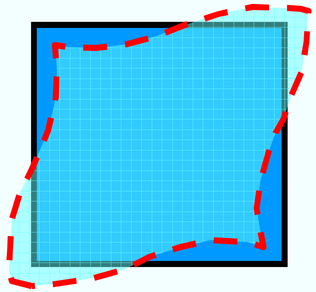
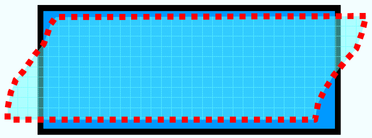
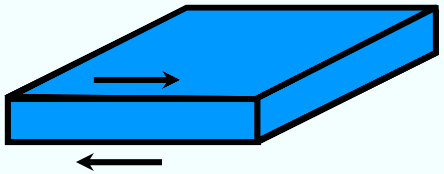
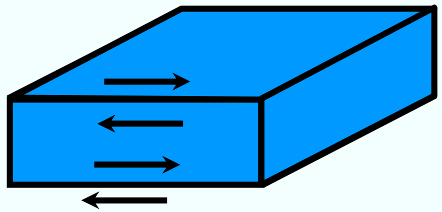
STRAIN		FIELD along: X	FIELD along: Y	FIELD along: Z
EXTENSIONAL along:	X	√		
	Y	√		
	Z			
SHEAR about:	X	√		
	Y		√	
	Z		√	



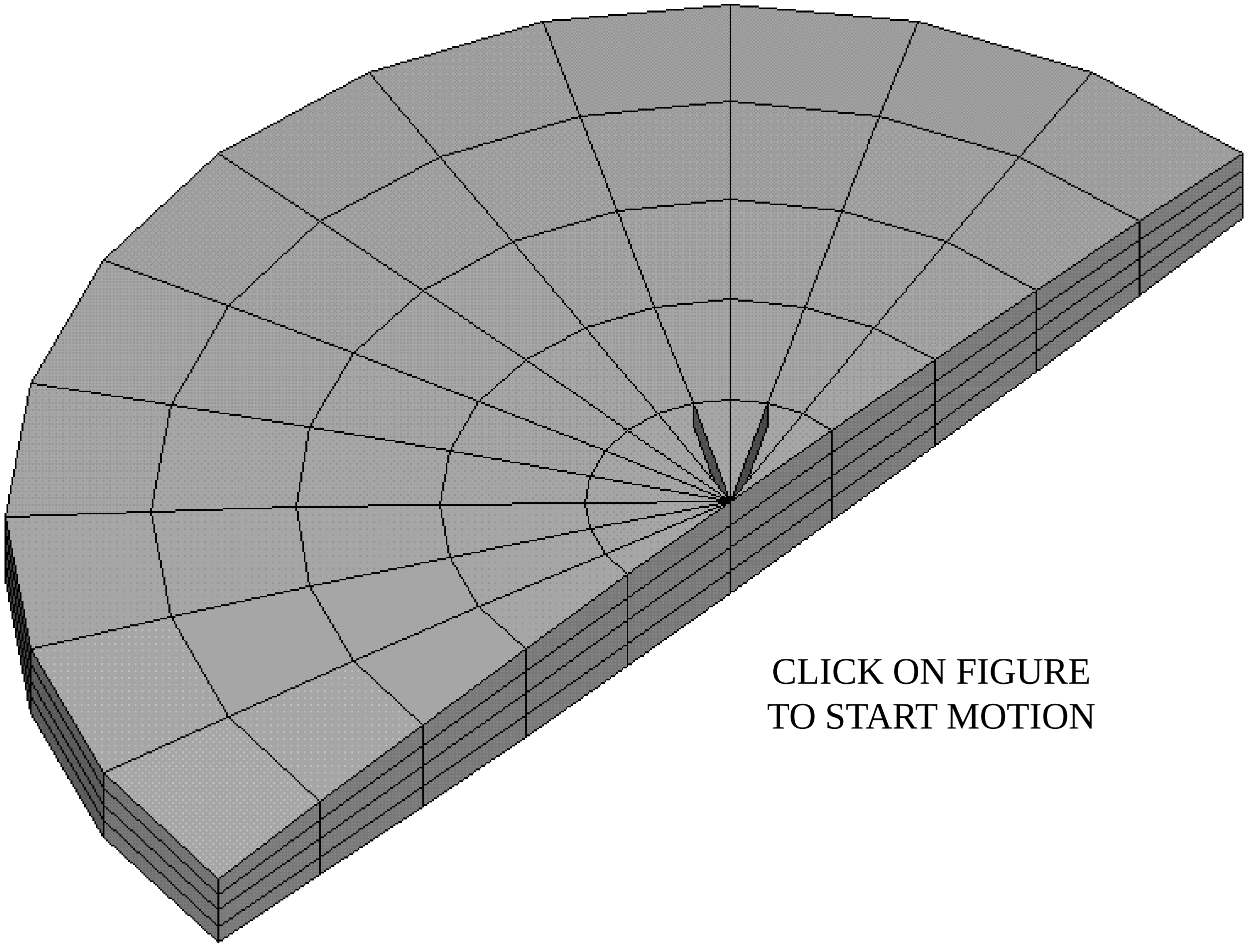
In quartz, the five strain components shown may be generated by an electric field. The modes shown on the next page may be excited by suitably placed and shaped electrodes. The shear strain about the Z-axis produced by the Y-component of the field is used in the rotated Y-cut family, including the AT, BT, and ST-cuts.

Modes of Motion

(Click on the mode names to see animation.)

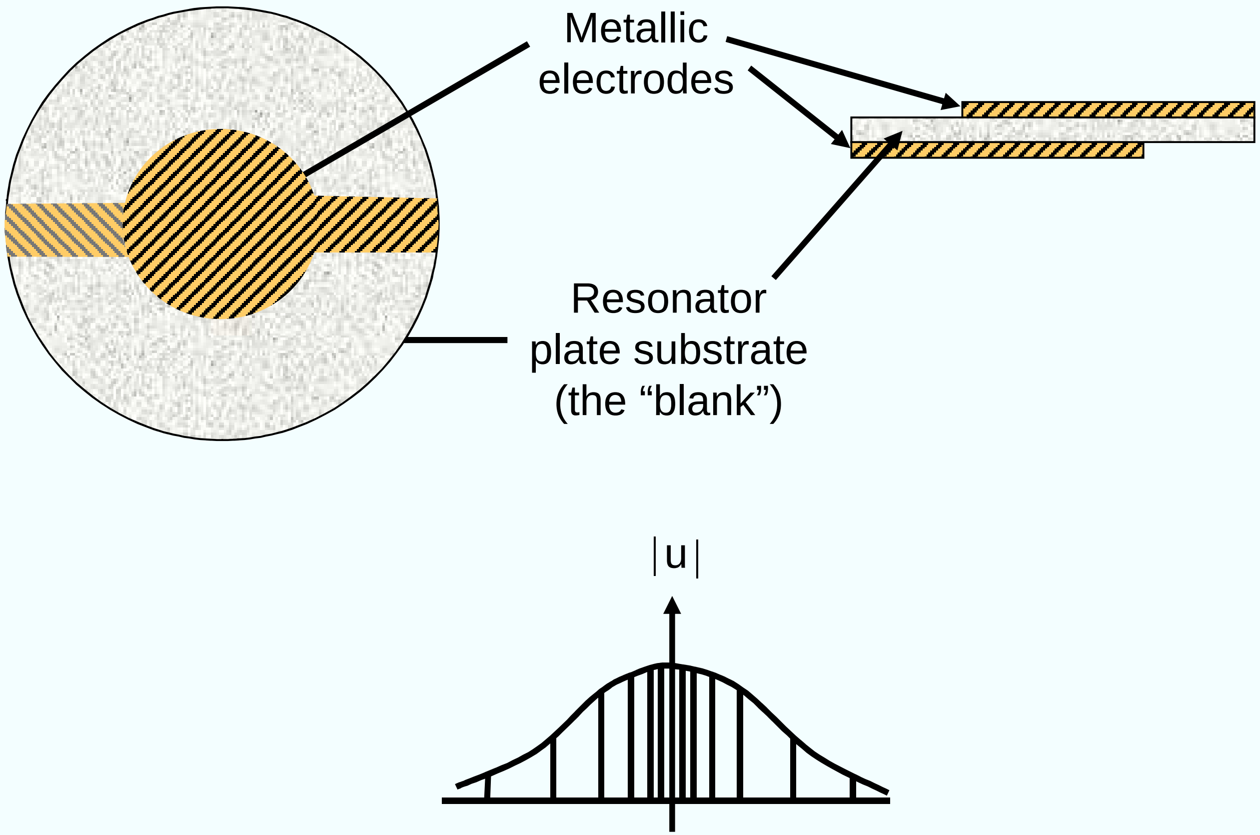
 Flexure Mode	 Extensional Mode	 Face Shear Mode
 Thickness Shear Mode	 Fundamental Mode Thickness Shear	 Third Overtone Thickness Shear

Motion Of A Thickness Shear Crystal



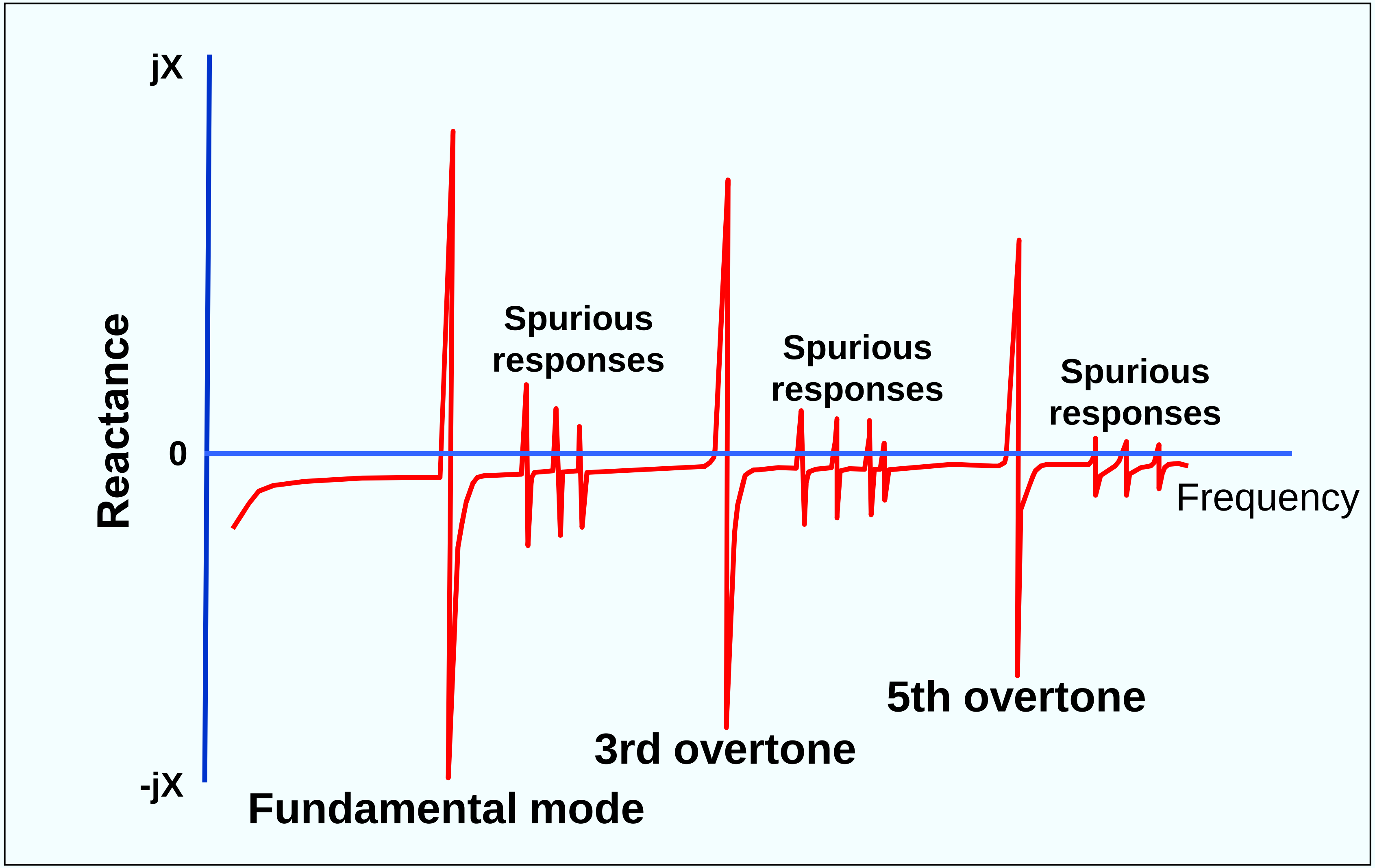
CLICK ON FIGURE
TO START MOTION

Resonator Vibration Amplitude Distribution



Conventional resonator geometry
and amplitude distribution, |u|

Overtone Response of a Quartz Crystal



Quartz is Highly Anisotropic

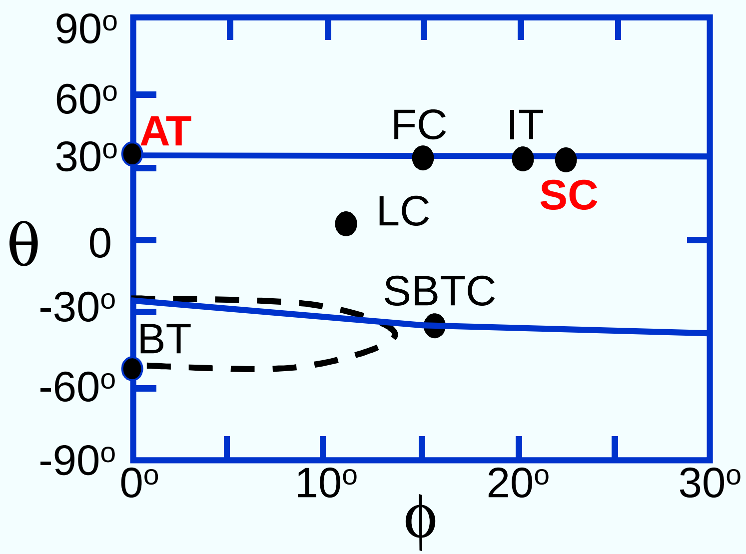
- The properties of quartz vary greatly with crystallographic direction. For example, when a quartz sphere is etched deeply in HF, the sphere takes on a triangular shape when viewed along the Z-axis, and a lenticular shape when viewed along the Y-axis. The etching rate is more than 100 times faster along the fastest etching rate direction (the Z-direction) than along the slowest direction (the slow-X-direction).

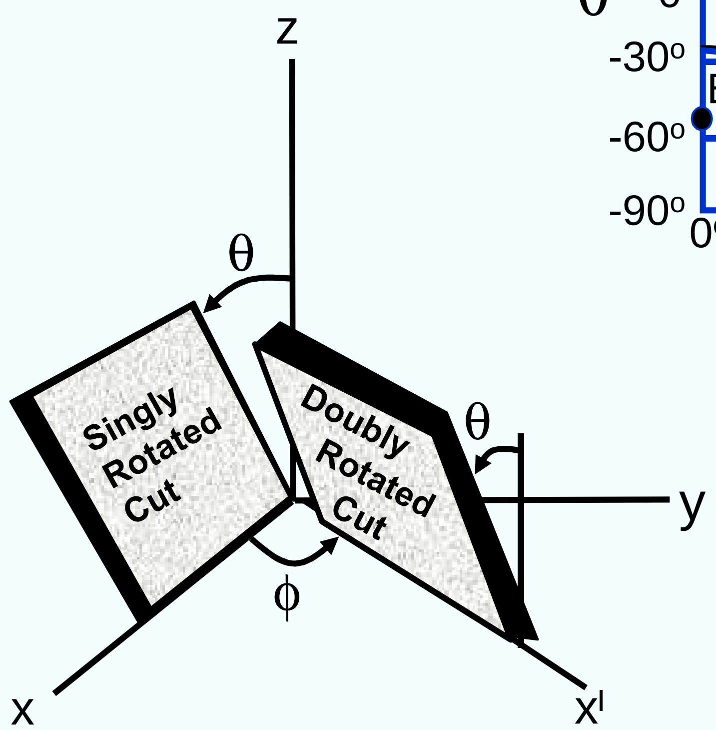
- The thermal expansion coefficient is 7.8 x 10^{-6}/°C along the Z-direction, and 14.3 x 10^{-6}/°C perpendicular to the Z-direction; the temperature coefficient of density is, therefore, -36.4 x 10^{-6}/°C.

- The temperature coefficients of the elastic constants range from -3300 x 10^{-6}/°C (for C_{12}) to +164 x 10^{-6}/°C (for C_{66}).

- For the proper angles of cut, the sum of the first two terms in T_f on the previous page is cancelled by the third term, i.e., temperature compensated cuts exist in quartz. (See next page.)

Zero Temperature Coefficient Quartz Cuts





The AT, FC, IT, SC, BT, and SBTC-cuts are some of the cuts on the locus of zero temperature coefficient cuts. The LC is a “linear coefficient” cut that has been used in a quartz thermometer.

Y-cut: ≈ +90 ppm/^{0}C
(thickness-shear mode)

X-cut: ≈ -20 ppm/^{0}C
(extensional mode)

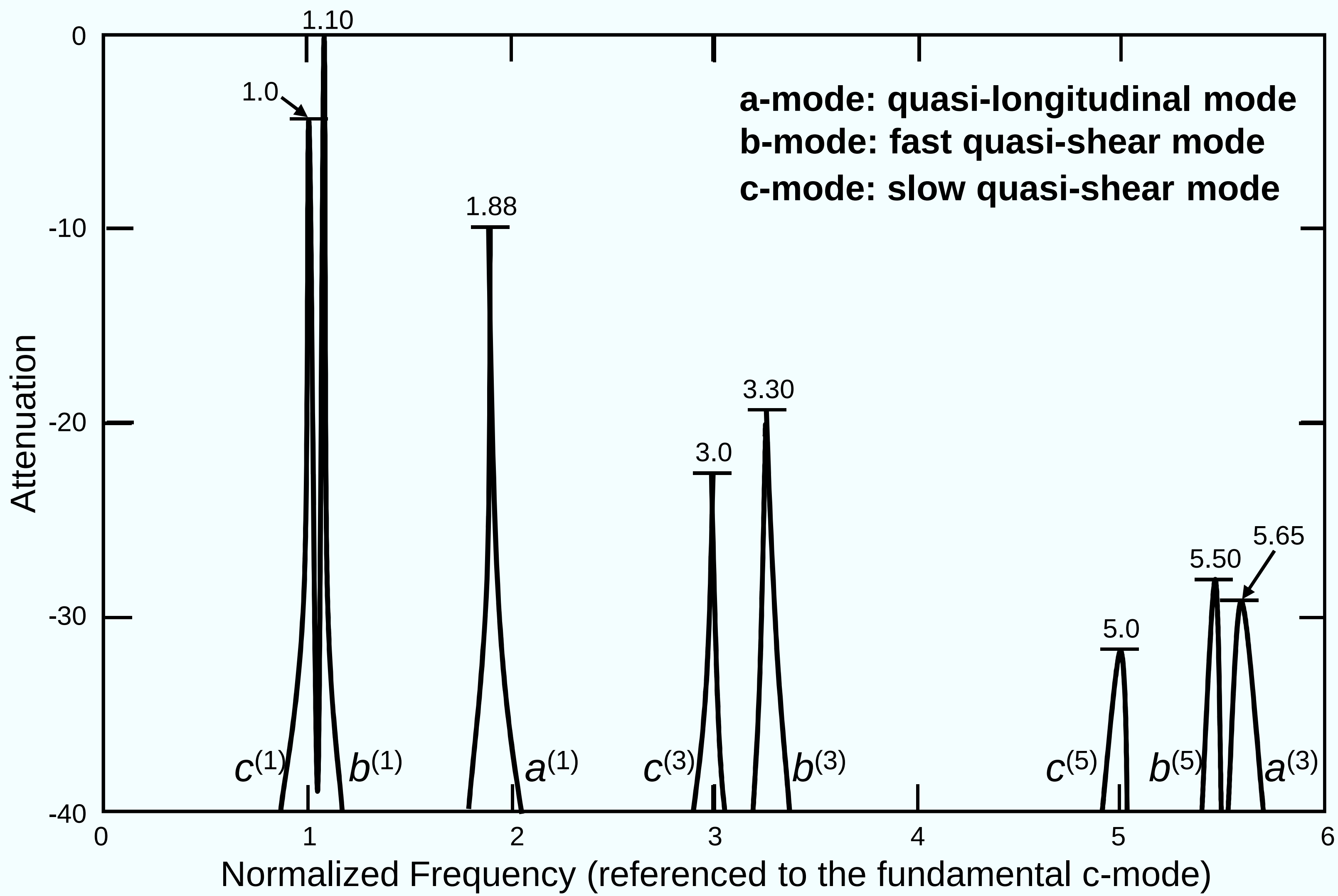
Comparison of SC and AT-cuts

- **Advantages of the SC-cut**
  - Thermal transient compensated (allows faster warmup OCXO)
  - Static and dynamic f vs. T allow higher stability OCXO and MCXO
  - Better f vs. T repeatability allows higher stability OCXO and MCXO
  - Far fewer activity dips
  - Lower drive level sensitivity
  - Planar stress compensated; lower Δf due to edge forces and bending
  - Lower sensitivity to radiation
  - Higher capacitance ratio (less Δf for oscillator reactance changes)
  - Higher Q for fundamental mode resonators of similar geometry
  - Less sensitive to plate geometry - can use wide range of contours

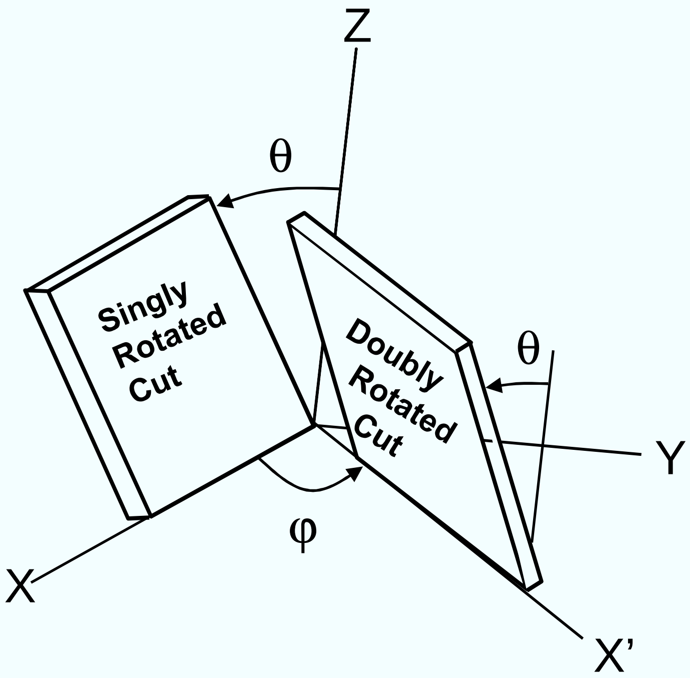
- **Disadvantage of the SC-cut :** More difficult to manufacture for OCXO (but is easier to manufacture for MCXO than is an AT-cut for precision TCXO)

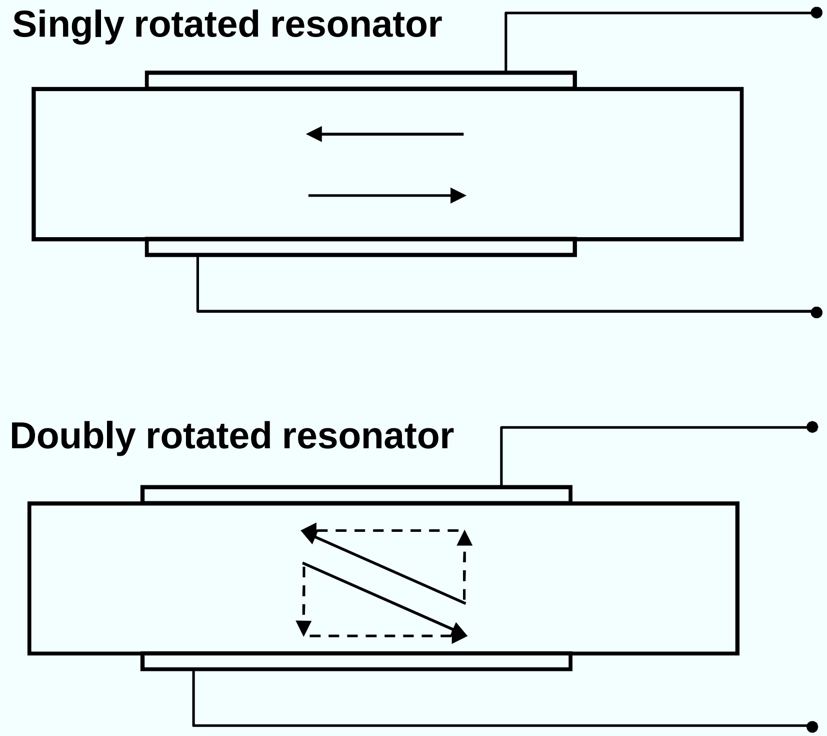
- **Other Significant Differences**
  - B-mode is excited in the SC-cut, although not necessarily in LFR's
  - The SC-cut is sensitive to electric fields (which can be used for compensation)

Mode Spectrograph of an SC-cut

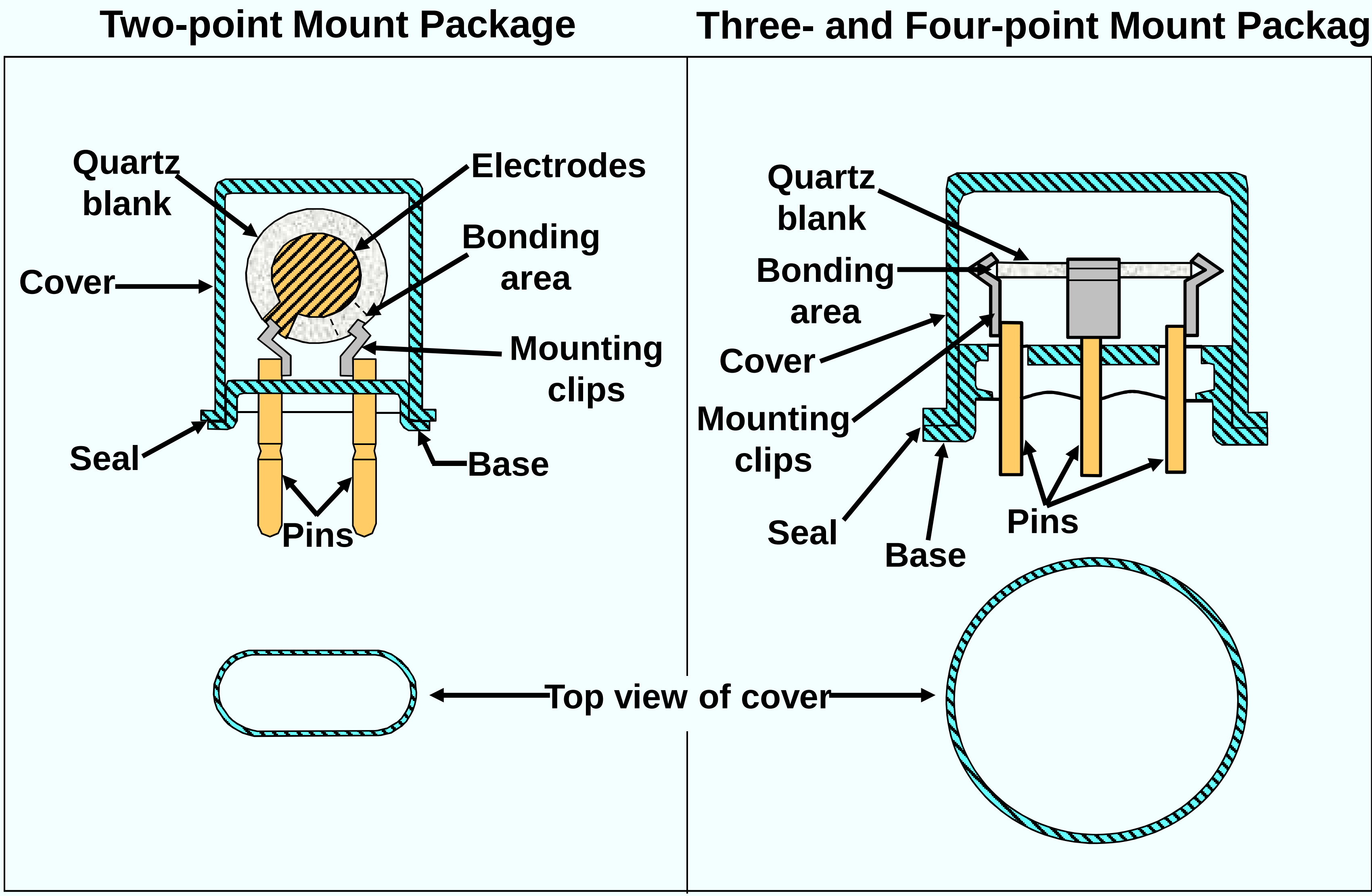


Singly Rotated and Doubly Rotated Cuts' Vibrational Displacements

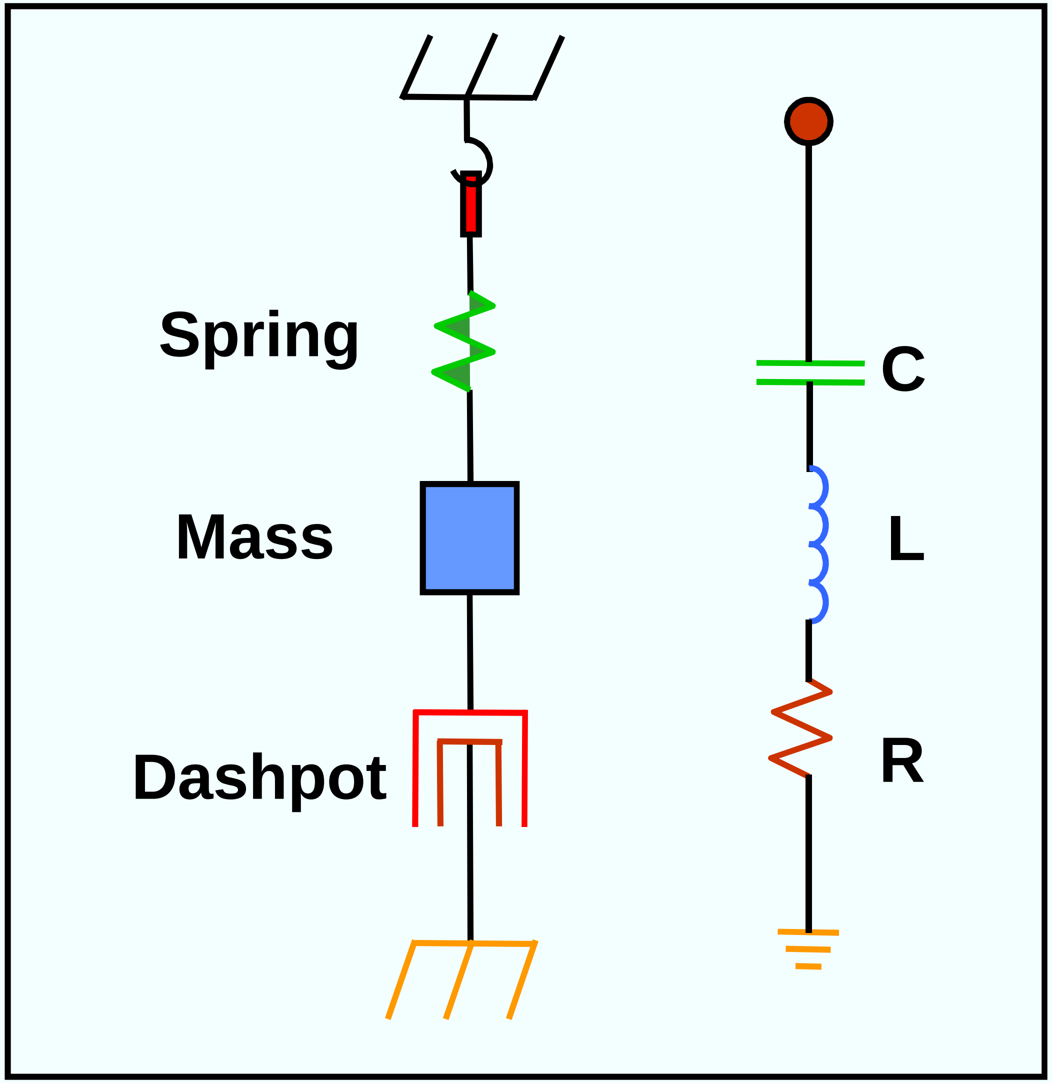




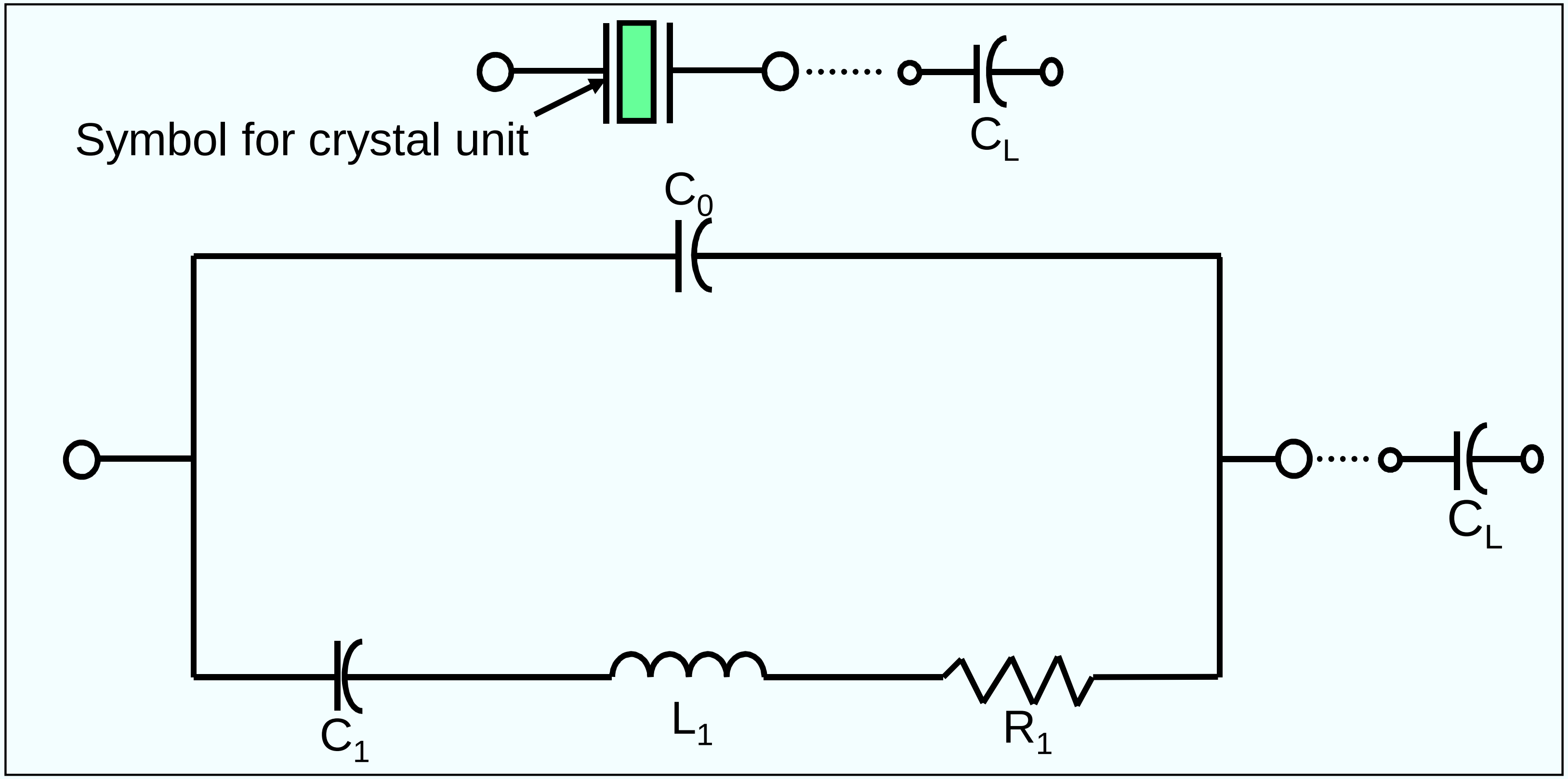
Resonator Packaging



Equivalent Circuits



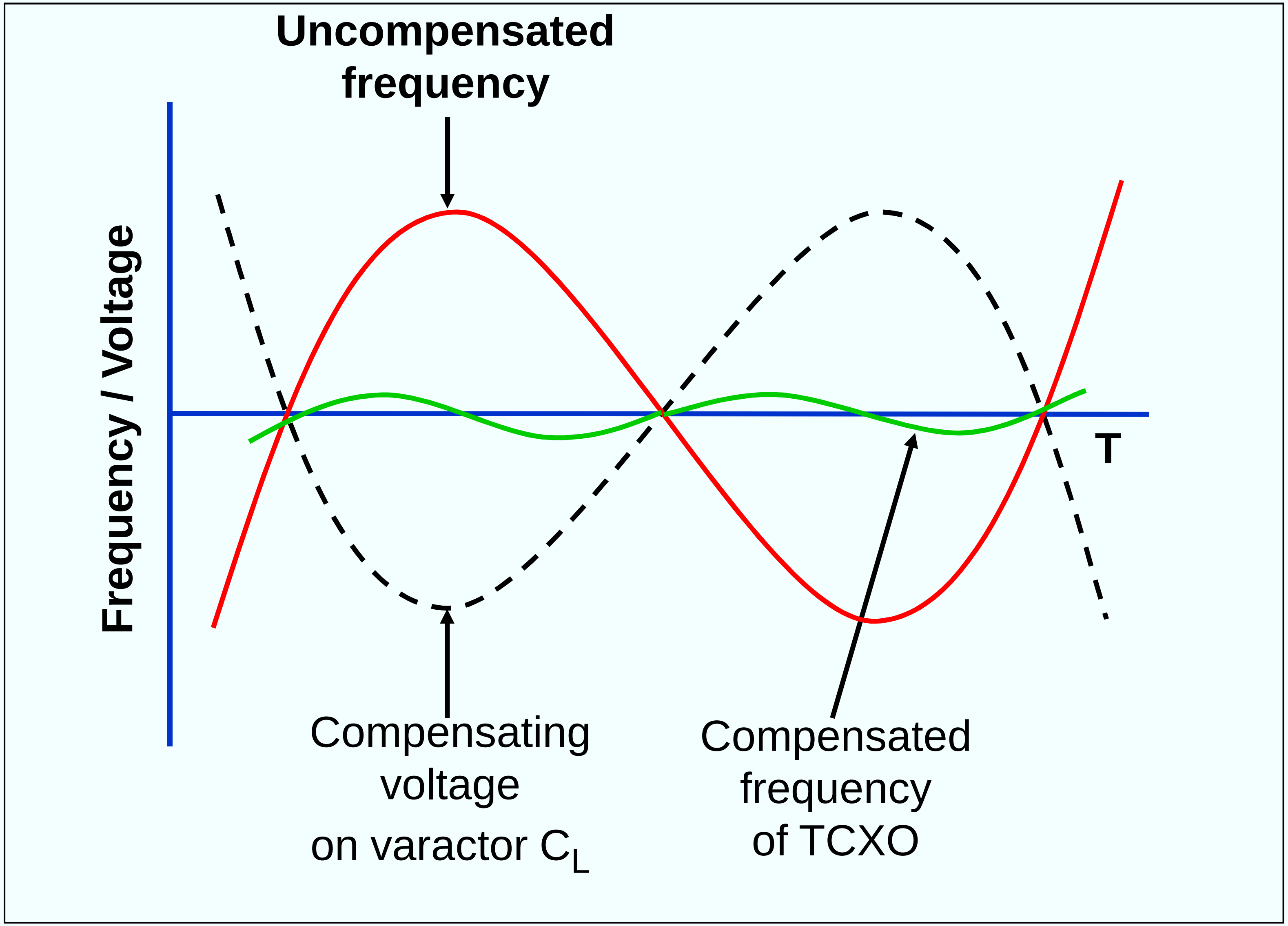
Equivalent Circuit of a Resonator



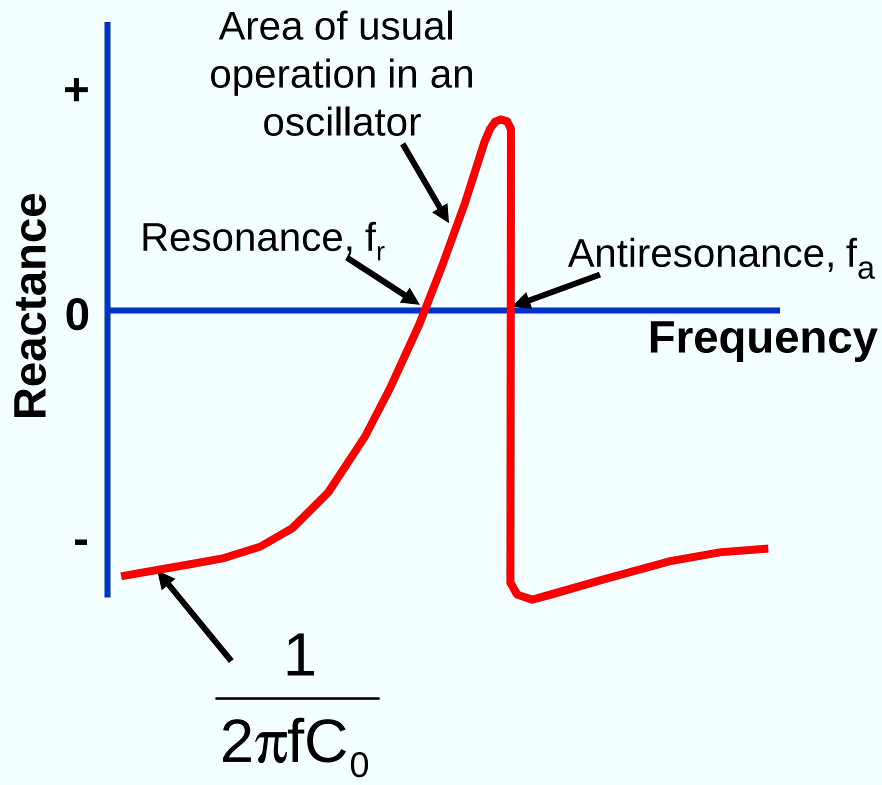
$$\frac{\Delta f}{f_S} \approx \frac{C_1}{2(C_0 + C_L)} \rightarrow$$

1. Voltage control (VCXO)
2. Temperature compensation (TCXO)

Crystal Oscillator f vs. T Compensation



Resonator Reactance vs. Frequency



Equivalent Circuit Parameter Relationships

$$C_0 \cong \varepsilon \frac{A}{t}$$

$$r \equiv \frac{C_0}{C_1}$$

$$f_S = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1}}$$

$$f_a - f_S \cong \frac{f_S}{2r}$$

$$Q = \frac{1}{2\pi f_S R_1 C_1}$$

$$\varphi = \frac{\omega L_1 - \frac{1}{\omega C_1}}{R_1}$$

$$\tau_1 = R_1 C_1 \cong 10^{-14}\,\mathrm{s}$$

$$\frac{d\varphi}{df} \cong \frac{360}{\pi}\frac{Q}{f_s}$$

$$C_{1n} \approx \frac{r' C_{11}}{n^3}$$

$$L_{1n} \approx \frac{n^3 L_{11}}{r'^3}$$

$$R_{1n} \approx \frac{n^3 R_{11}}{r'}$$

$$2r = \left(\frac{\pi n}{2k}\right)^2$$

n: Overtone number
C_0: Static capacitance
C_1: Motional capacitance
C_{1n}: C_1 of n-th overtone
L_1: Motional inductance
L_{1n}: L_1 of n-th overtone
R_1: Motional resistance
R_{1n}: R_1 of n-th overtone
ε: Dielectric permittivity of quartz $\approx$40 pF/m (average)
A: Electrode area
t: Plate thickness
r: Capacitance ratio
r': f_n/f_1
f_s: Series resonance frequency $\approx f_R$
f_a: Antiresonance frequency
Q; Quality factor
τ_1: Motional time constant
ω: Angular frequency = $2\pi f$
φ: Phase angle of the impedance
k; Piezoelectric coupling factor =8.8% for AT-cut, 4.99% for SC

What is Q and Why is it Important?

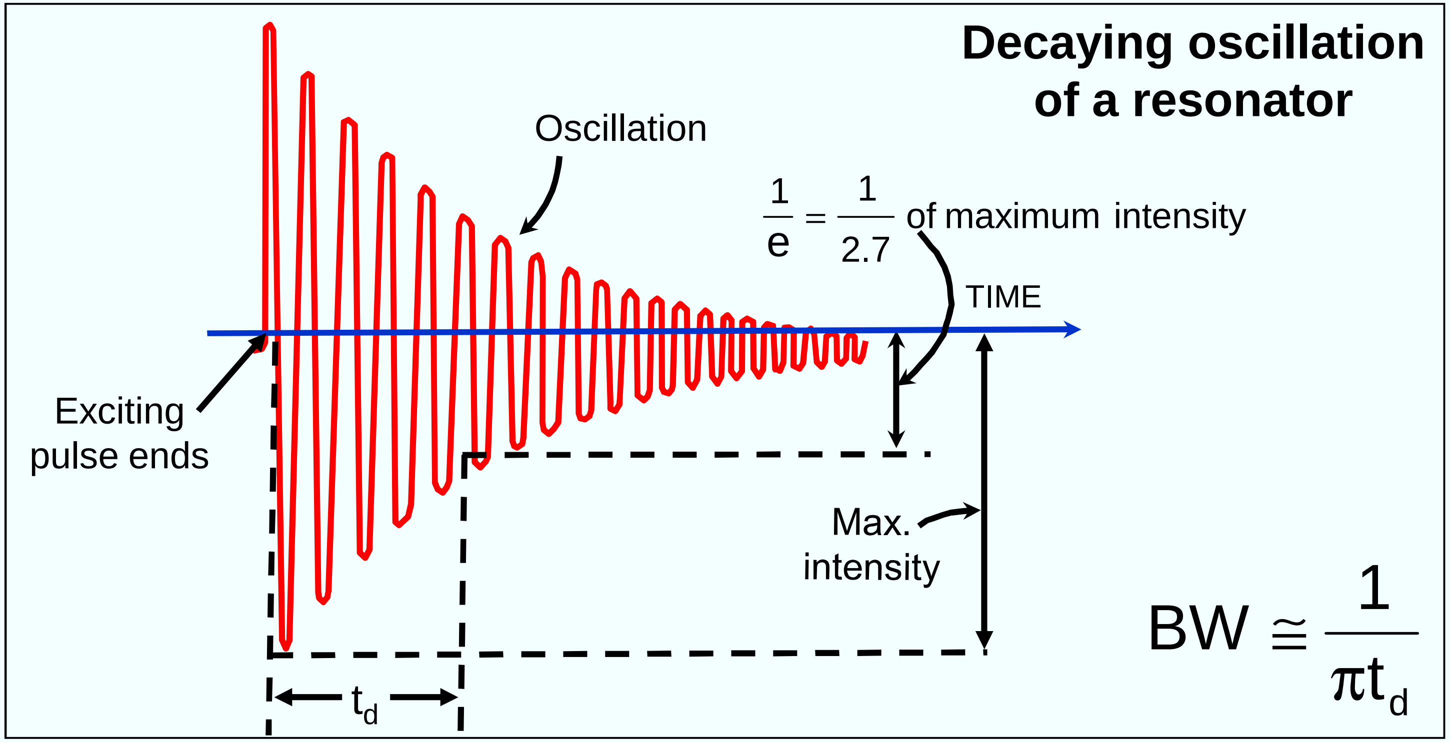
$$Q \equiv 2\pi \frac{\text{Energy stored during a cycle}}{\text{Energy dissipated per cycle}}$$

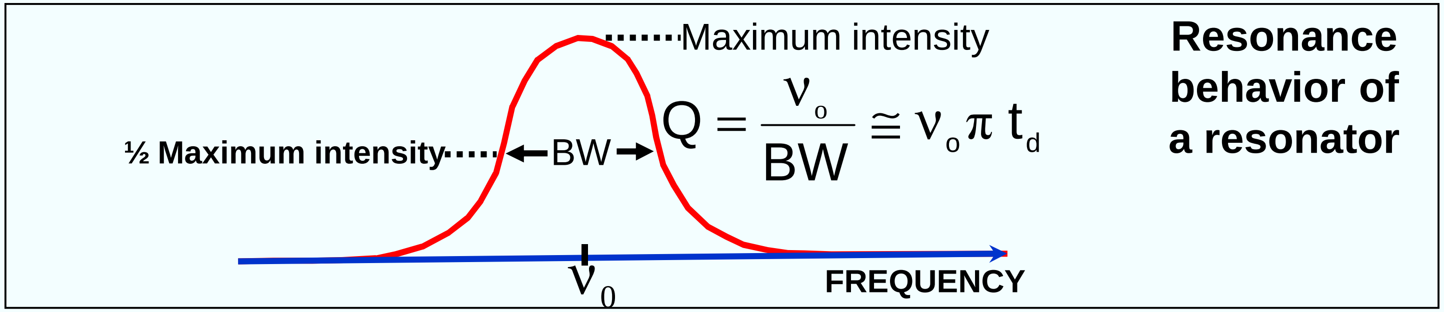
Q is proportional to the decay-time, and is inversely proportional to the linewidth of resonance (see next page).

- The higher the Q, the higher the frequency stability and accuracy **capability** of a resonator (i.e., high Q is a necessary but not a sufficient condition). If, e.g., $Q = 10^6$, then 10^{-10} accuracy requires ability to determine center of resonance curve to 0.01% of the linewidth, and stability (for some averaging time) of 10^{-12} requires ability to stay near peak of resonance curve to 10^{-6} of linewidth.

- Phase noise close to the carrier has an especially strong dependence on Q ($\mathcal{L}(f) \propto 1/Q^4$ for quartz oscillators).

Decay Time, Linewidth, and Q





Factors that Determine Resonator Q

The **maximum Q** of a resonator can be expressed as:

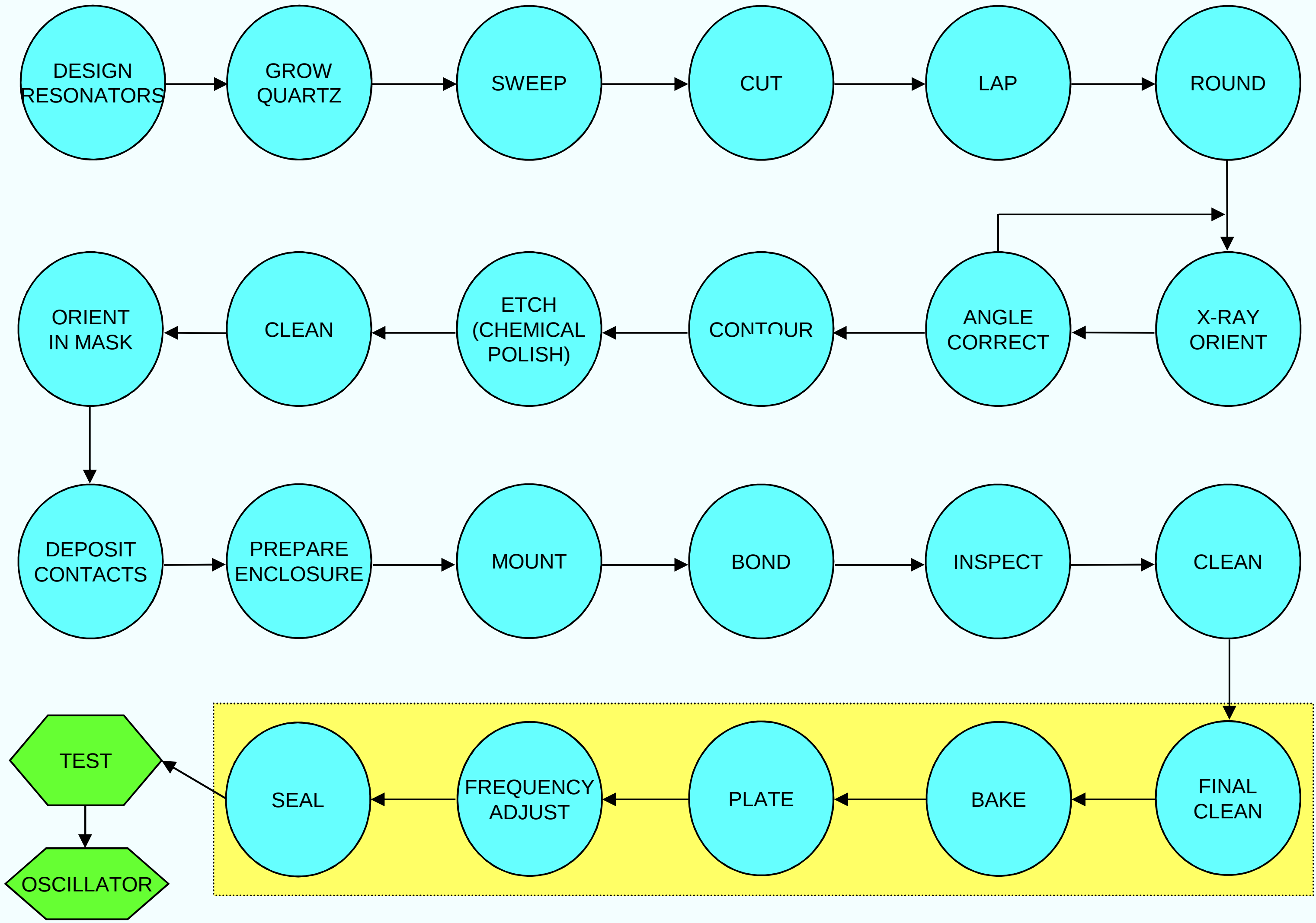
$$Q_{max} = \frac{1}{2\pi f\tau} ,$$

where f is the frequency in Hz, and τ is an empirically determined “motional time constant” in seconds, which varies with the angles of cut and the mode of vibration. For example, $\tau = 1 \times 10^{-14}$s for the AT-cut's c-mode (Q_{max} = 3.2 million at 5 MHz), $\tau = 9.9 \times 10^{-15}$s for the SC-cut's c-mode, and $\tau = 4.9 \times 10^{-15}$s for the BT-cut's b-mode.

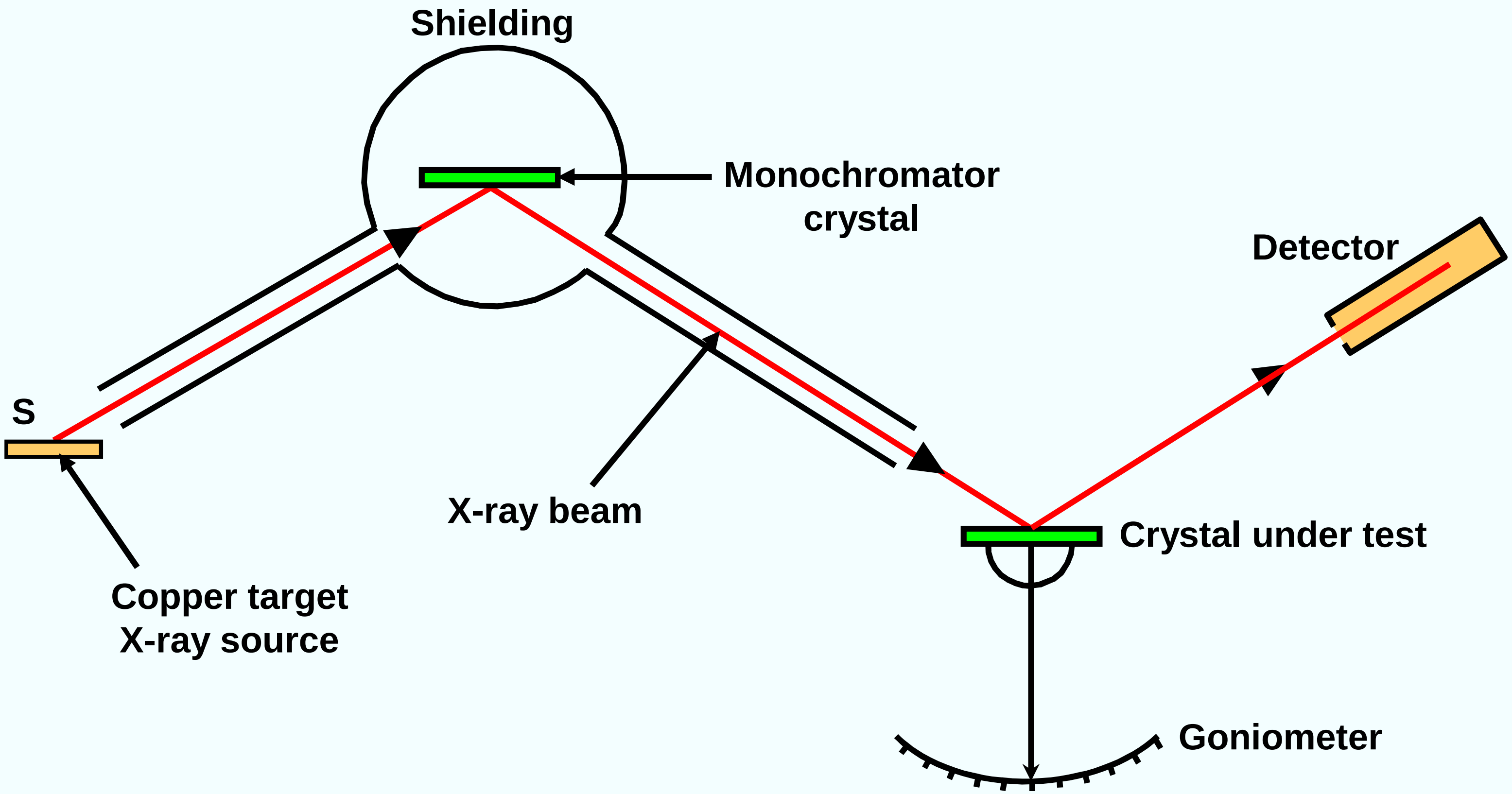
Other factors which affect the Q of a resonator include:

- Overtone
- Surface finish
- Material impurities and defects
- Mounting stresses
- Bonding stresses
- Temperature
- Electrode geometry and type
- Blank geometry (contour, dimensional ratios)
- Drive level
- Gases inside the enclosure (pressure, type of gas)
- Interfering modes
- Ionizing radiation

Resonator Fabrication Steps



X-ray Orientation of Crystal Plates



Double-crystal x-ray diffraction system

Contamination Control

Contamination control is essential during the fabrication of resonators because contamination can adversely affect:

- Stability (see chapter 4)
  - aging
  - hysteresis
  - retrace
  - noise
  - nonlinearities and resistance anomalies (high starting resistance, second-level of drive, intermodulation in filters)
  - frequency jumps?
- Manufacturing yields
- Reliability

Crystal Enclosure Contamination

The enclosure and sealing process can have important influences on resonator stability.

- A monolayer of adsorbed contamination contains ~ 10^{15} molecules/cm^2 (on a smooth surface)
- An enclosure at 10^{-7} torr contains ~10^9 gaseous molecules/cm^3

Therefore:

In a 1 cm^3 enclosure that has a monolayer of contamination on its inside surfaces, there are ~10^6 times more adsorbed molecules than gaseous molecules when the enclosure is sealed at 10^{-7} torr. The desorption and adsorption of such adsorbed molecules leads to aging, hysteresis, retrace, noise, etc.

Milestones in Quartz Technology

1880	Piezoelectric effect discovered by Jacques and Pierre Curie
1905	First hydrothermal growth of quartz in a laboratory - by G. Spezia
1917	First application of piezoelectric effect, in sonar
1918	First use of piezoelectric crystal in an oscillator
1926	First quartz crystal controlled broadcast station
1927	First temperature compensated quartz cut discovered
1927	First quartz crystal clock built
1934	First practical temp. compensated cut, the AT-cut, developed
1949	Contoured, high-Q, high stability AT-cuts developed
1956	First commercially grown cultured quartz available
1956	First TCXO described
1972	Miniature quartz tuning fork developed; quartz watches available
1974	The SC-cut (and TS/TTC-cut) predicted; verified in 1976
1982	First MCXO with dual c-mode self-temperature sensing

Quartz Resonators for Wristwatches

Requirements:

- Small size
- Low power dissipation (including the oscillator)
- Low cost
- High stability (temperature, aging, shock, attitude)

These requirements can be met with 32,768 Hz quartz tuning forks

Why 32,768 Hz?

$32{,}768 = 2^{15}$

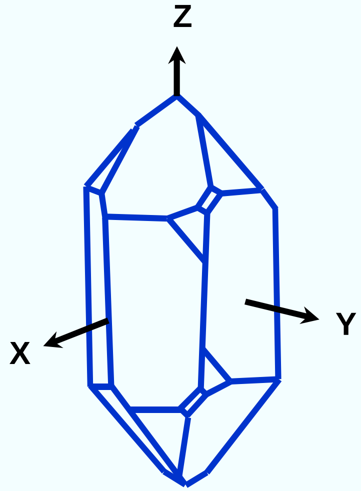
- In an analog watch, a stepping motor receives one impulse per second which advances the second hand by 6°, i.e., 1/60th of a circle, every second.

- Dividing 32,768 Hz by two 15 times results in 1 Hz.

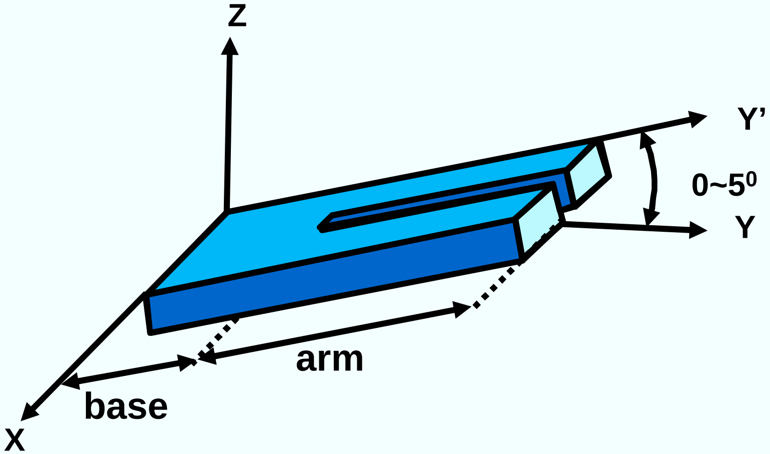
- The 32,768 Hz is a compromise among size, power requirement (i.e., battery life) and stability.

32,768
16,384
8,192
4,096
2,048
1,024
512
256
128
64
32
16
8
4
2
1

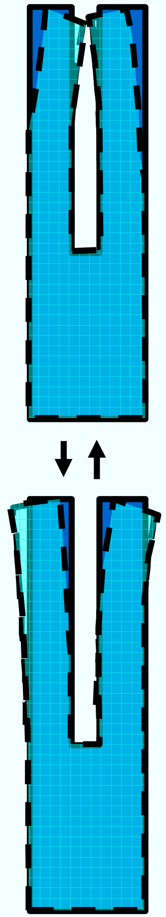
Quartz Tuning Fork



a) natural faces and crystallographic axes of quartz

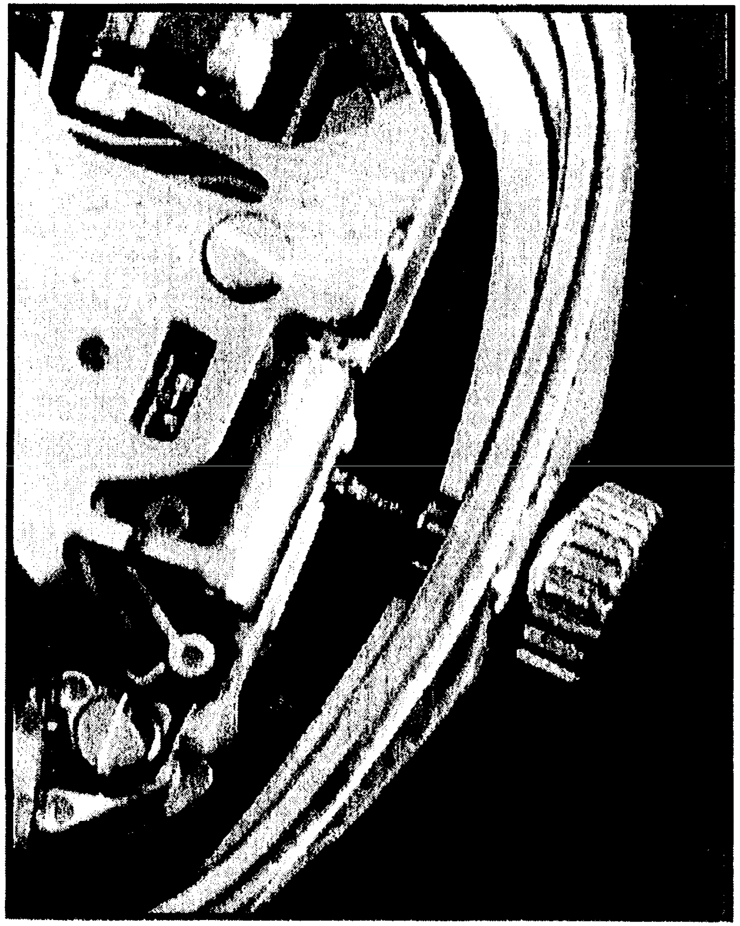


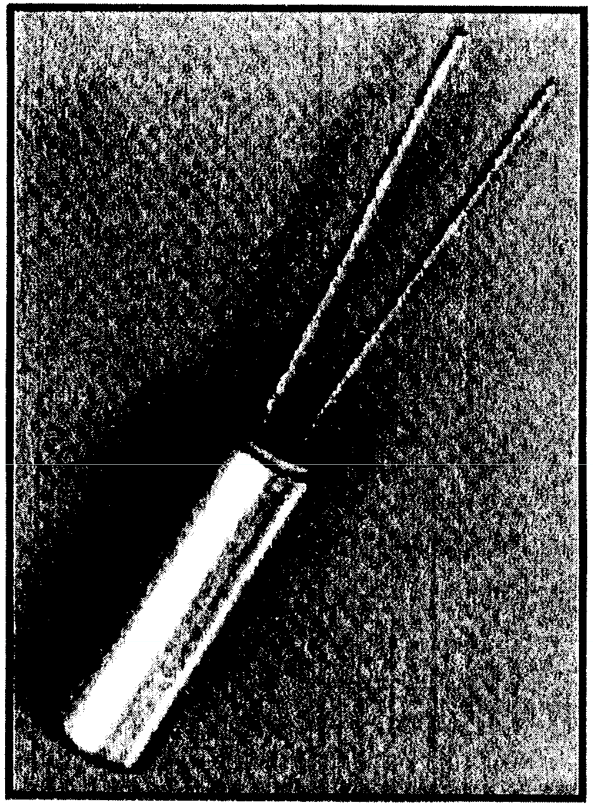
b) crystallographic orientation of tuning fork



c) vibration mode of tuning fork

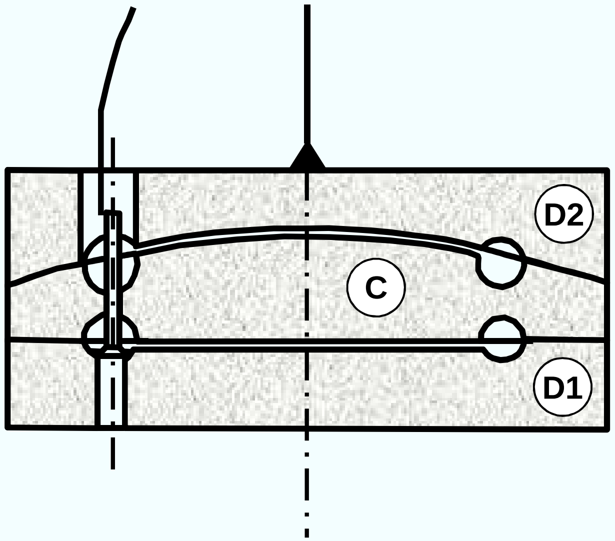
Watch Crystal



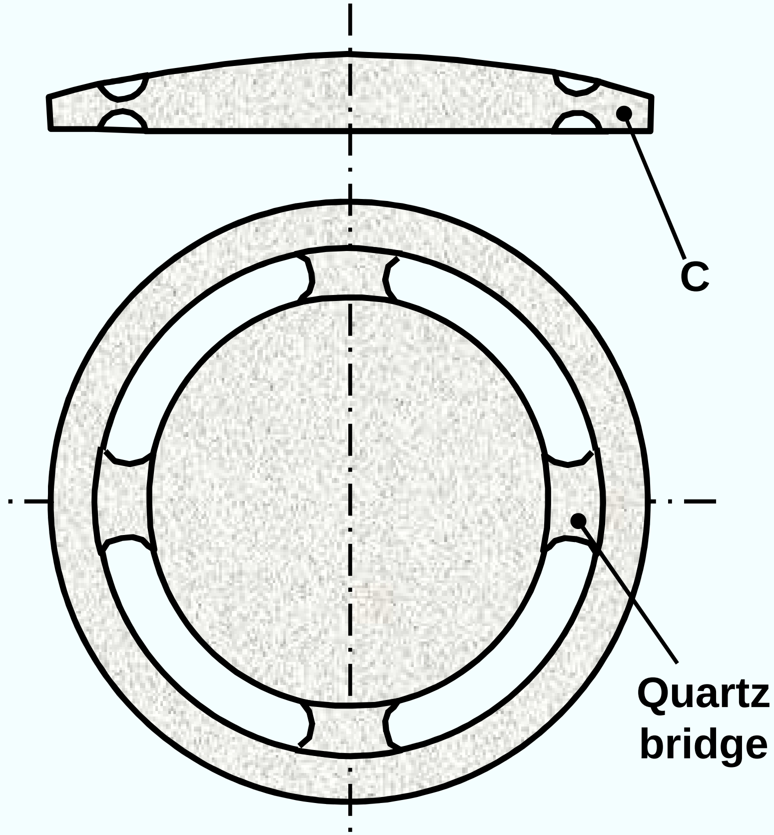


2 mm diameter x 6 mm tall cylinder
(0.08" diameter x 0.24" tall)

Electrodeless (BVA) Resonator



Side view of BVA_2
resonator construction

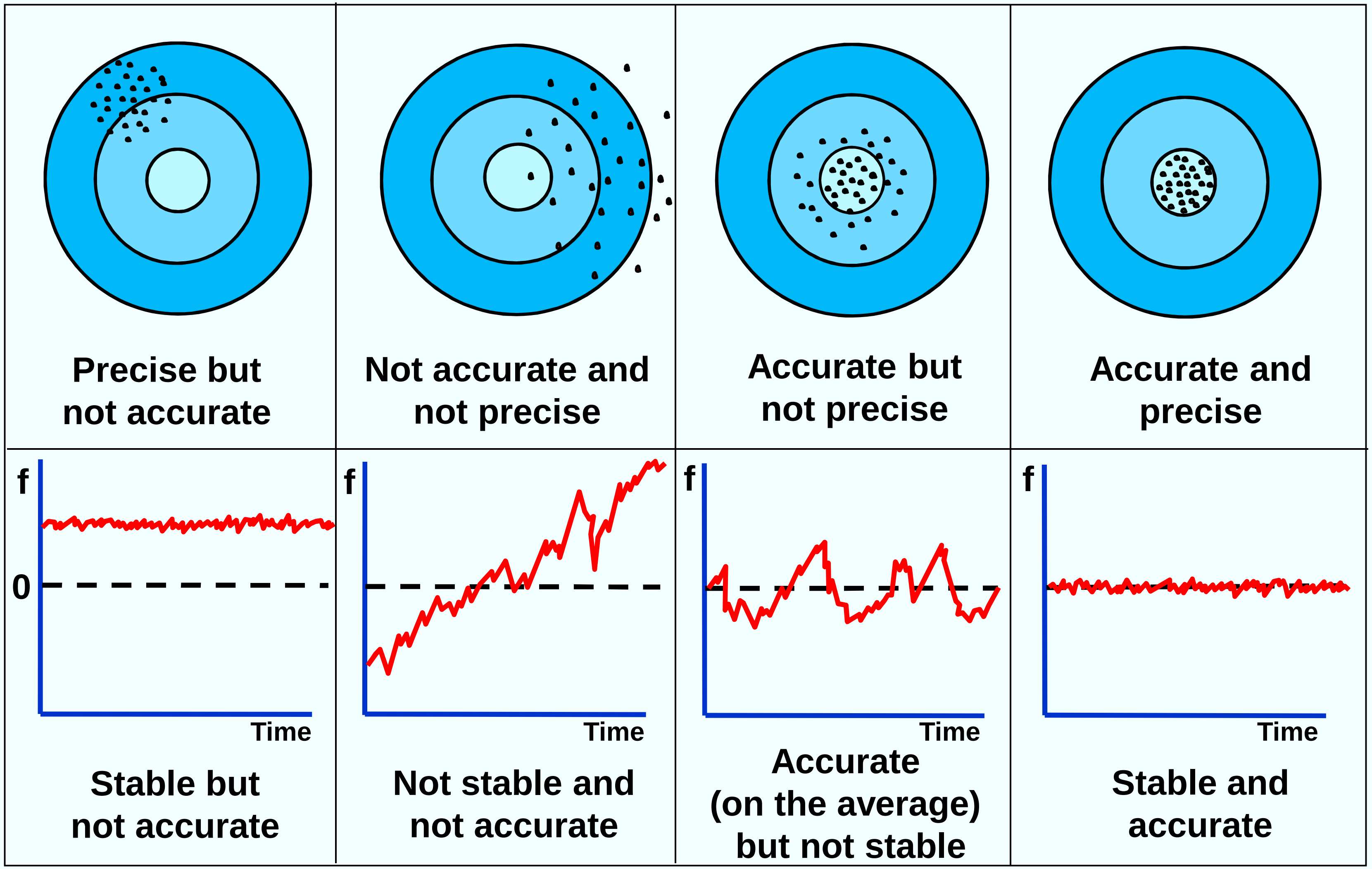


Side and top views of
center plate C

The Units of Stability in Perspective

- What is one part in 10^{10} ? (As in 1 x 10^{-10}/day aging.)
  - ~1/2 cm out of the circumference of the earth.
  - ~1/4 second per human lifetime (of ~80 years).
- Power received on earth from a GPS satellite, -160 dBW, is as “bright” as a flashlight in Los Angeles would look in New York City, ~5000 km away (neglecting earth’s curvature).
- What is -170 dB? (As in -170 dBc/Hz phase noise.)
  - -170 dB = 1 part in 10^{17} ≈ thickness of a sheet of paper out of the total distance traveled by all the cars in the world in a day.

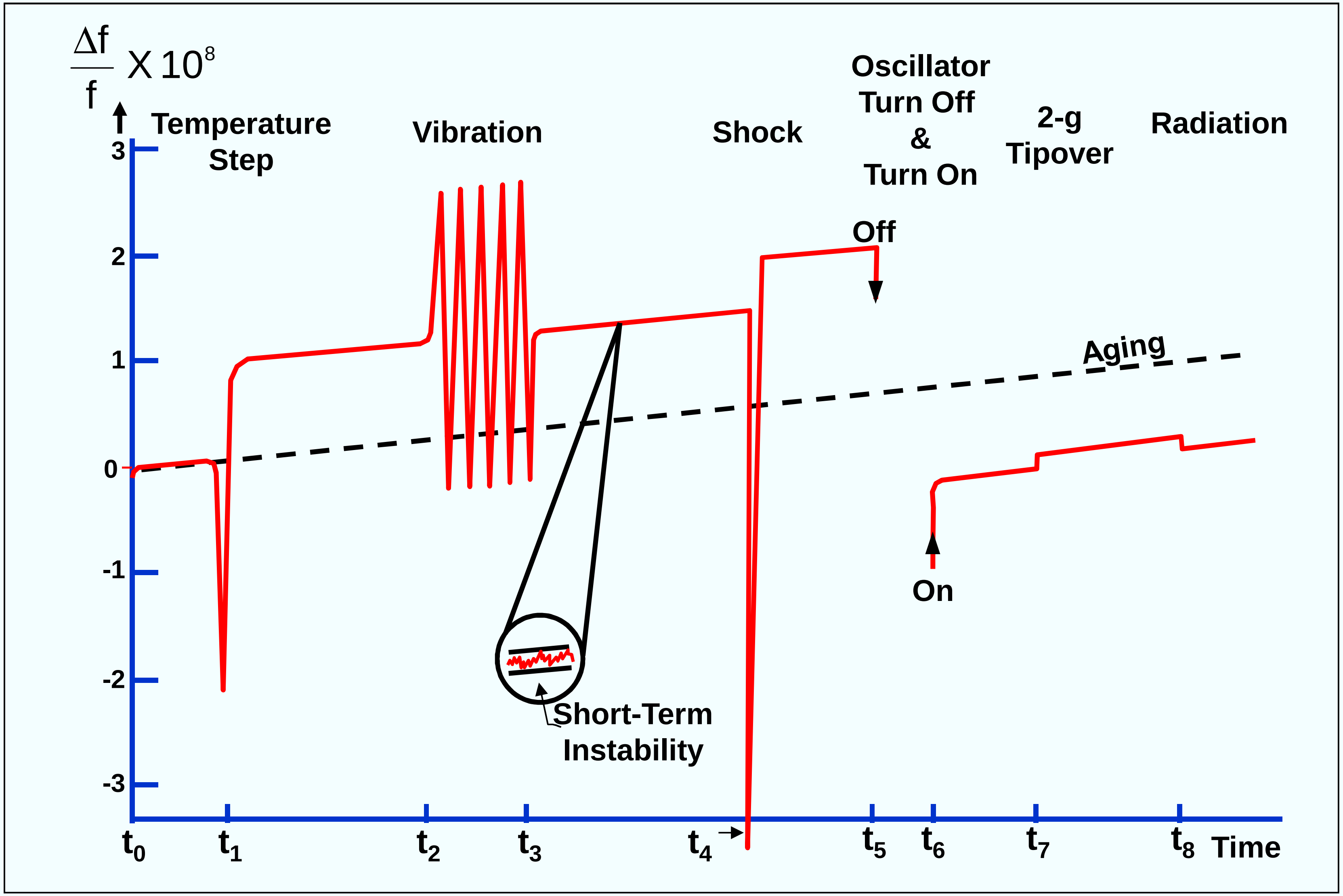
Accuracy, Precision, and Stability



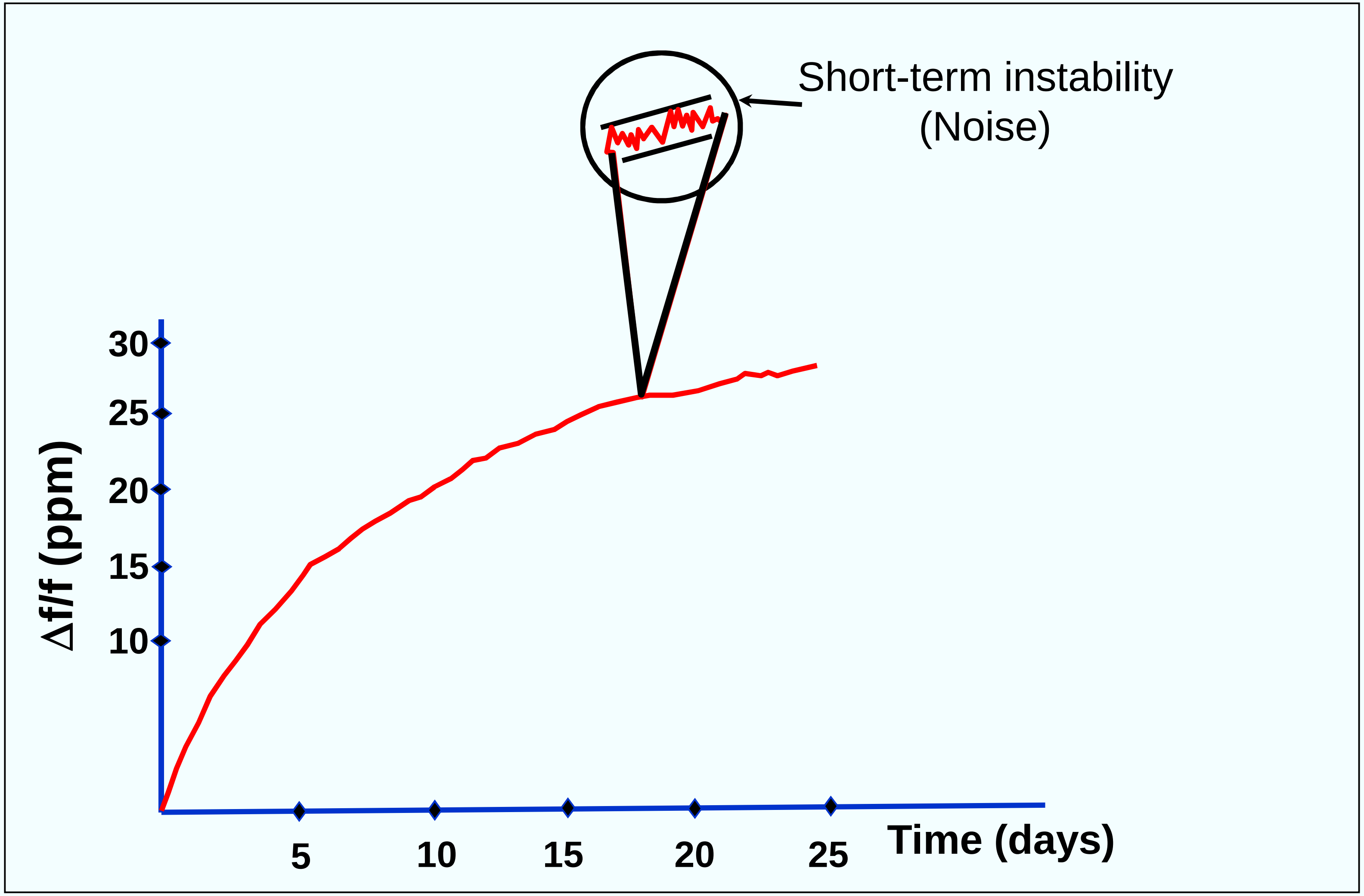
Influences on Oscillator Frequency

- **Time**
  - Short term (noise)
  - Intermediate term (e.g., due to oven fluctuations)
  - Long term (aging)
- **Temperature**
  - Static frequency vs. temperature
  - Dynamic frequency vs. temperature (warmup, thermal shock)
  - Thermal history ("hysteresis," "retrace")
- **Acceleration**
  - Gravity (2g tipover)
  - Vibration
  - Acoustic noise
  - Shock
- **Ionizing radiation**
  - Steady state
  - Pulsed
  - Photons (X-rays, γ-rays)
  - Particles (neutrons, protons, electrons)
- **Other**
  - Power supply voltage
  - Atmospheric pressure (altitude)
  - Humidity
  - Load impedance
  - Magnetic field

Idealized Frequency-Time-Influence Behavior



Aging and Short-Term Stability



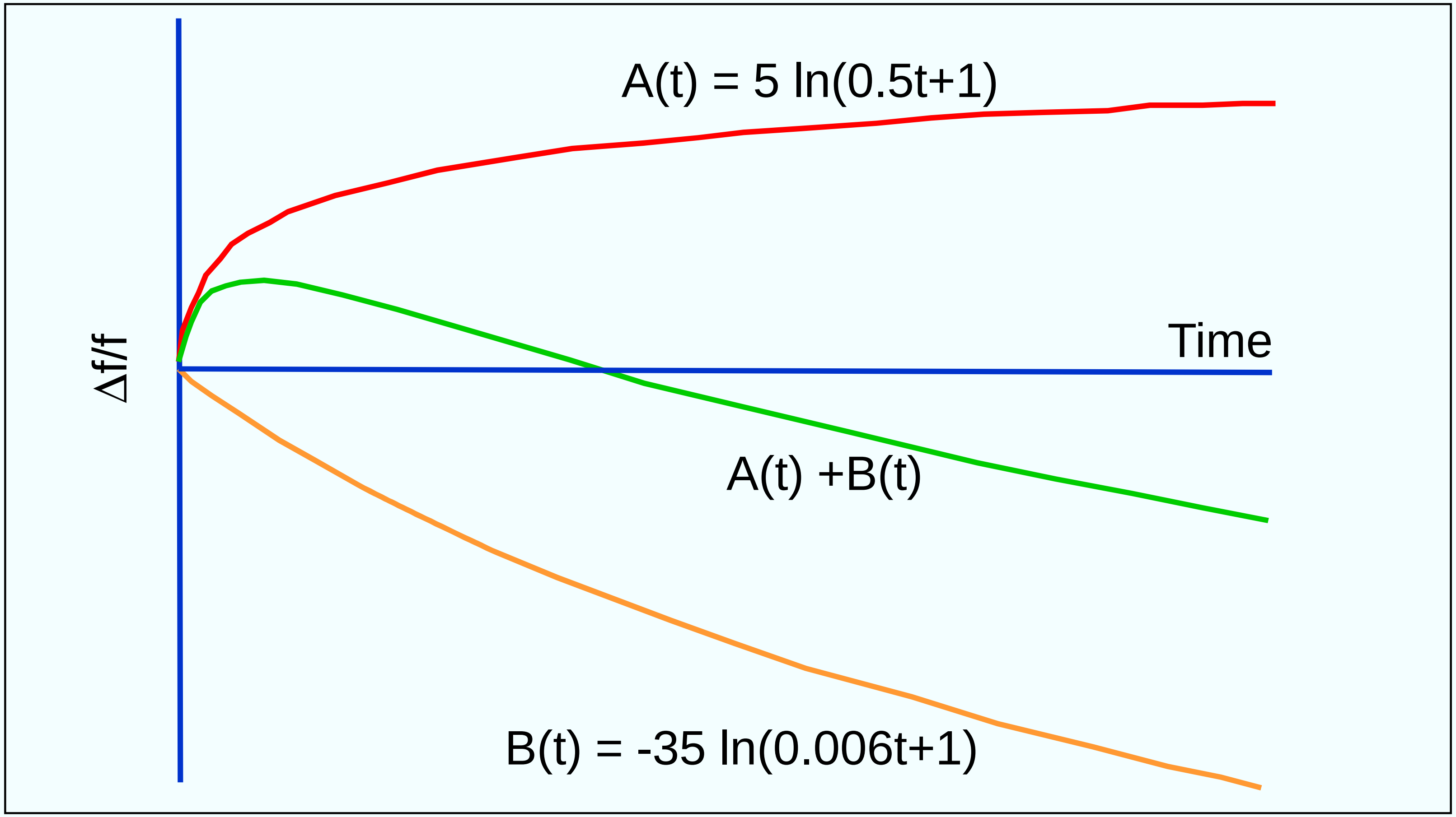
Aging Mechanisms

- **Mass transfer due to contamination**
  Since $f \propto 1/t$, $\Delta f/f = -\Delta t/t$; e.g., $f_{5MHz\ Fund} \approx 10^6$ molecular layers, therefore, 1 quartz-equivalent monolayer $\Rightarrow \Delta f/f \approx 1$ ppm

- **Stress relief** in the resonator's: mounting and bonding structure, electrodes, and in the quartz (?)

- **Other effects**
  - Quartz outgassing
  - Diffusion effects
  - Chemical reaction effects
  - Pressure changes in resonator enclosure (leaks and outgassing)
  - Oscillator circuit aging (load reactance and drive level changes)
  - Electric field changes (doubly rotated crystals only)
  - Oven-control circuitry aging

Typical Aging Behaviors



Stresses on a Quartz Resonator Plate

Causes:

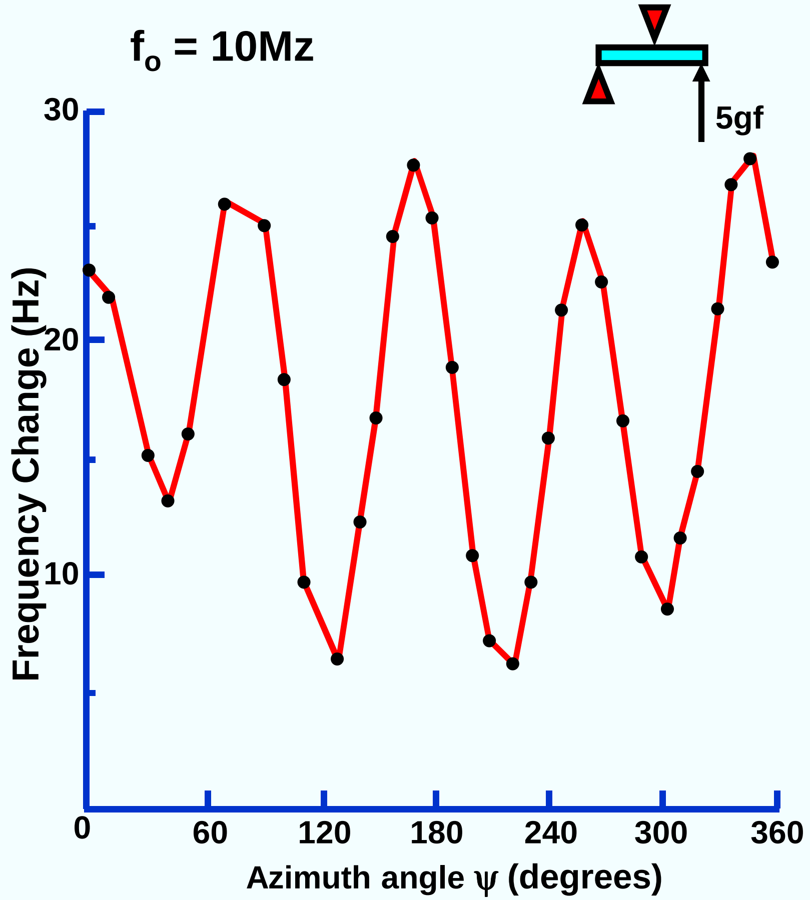
- Thermal expansion coefficient differences
- Bonding materials changing dimensions upon solidifying/curing
- Residual stresses due to clip forming and welding operations, sealing
- Intrinsic stresses in electrodes
- Nonuniform growth, impurities & other defects during quartz growing
- Surface damage due to cutting, lapping and (mechanical) polishing

Effects:

- In-plane diametric forces
- Tangential (torsional) forces, especially in 3 and 4-point mounts
- Bending (flexural) forces, e.g., due to clip misalignment and electrode stresses
- Localized stresses in the quartz lattice due to dislocations, inclusions, other impurities, and surface damage

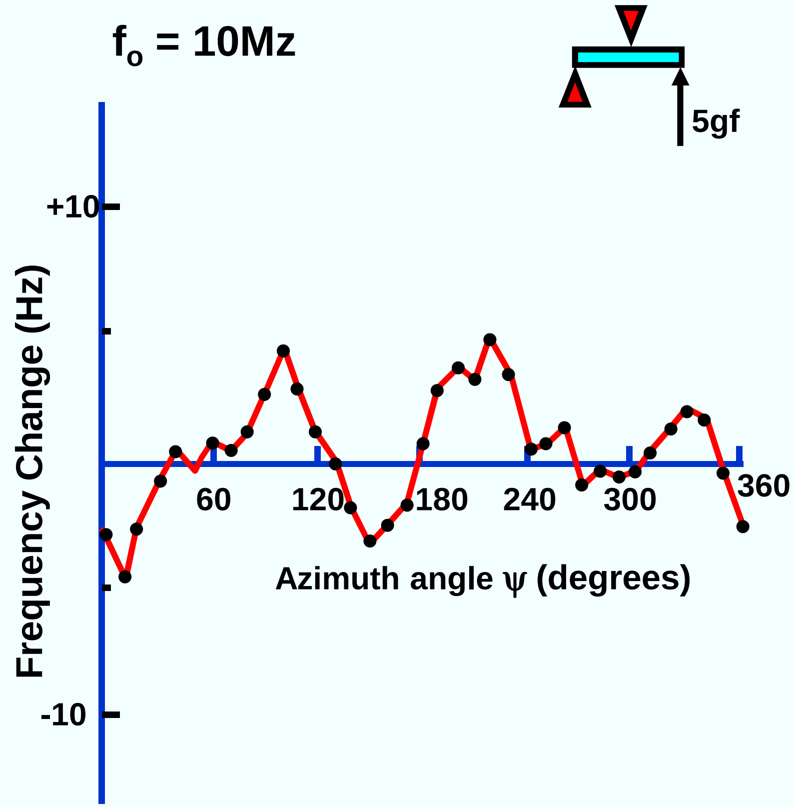
Bending Force vs. Frequency Change

AT-cut resonator



Frequency change for symmetrical bending, AT-cut crystal.

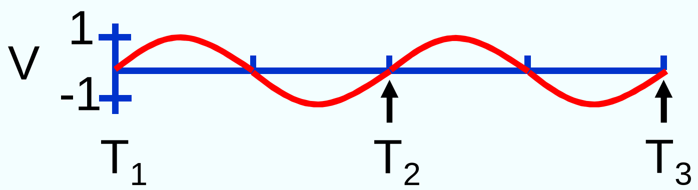
SC-cut resonator



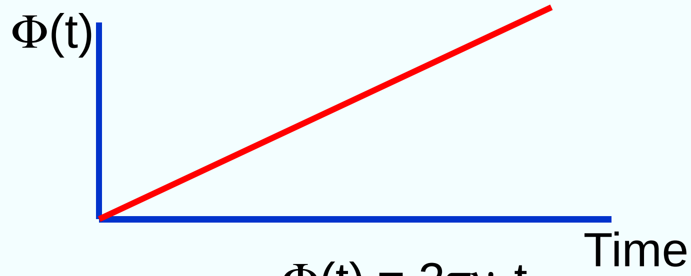
Frequency change for symmetrical bending, SC-cut crystal.

Short Term Instability (Noise)

Stable Frequency (Ideal Oscillator)

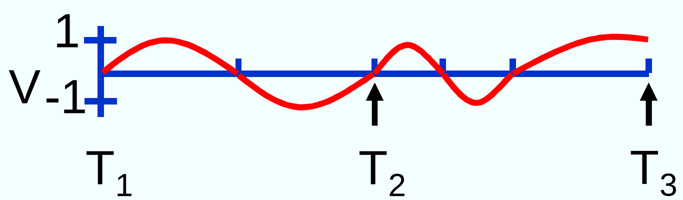


$V(t) = V_0 \sin(2\pi\nu_0 t)$

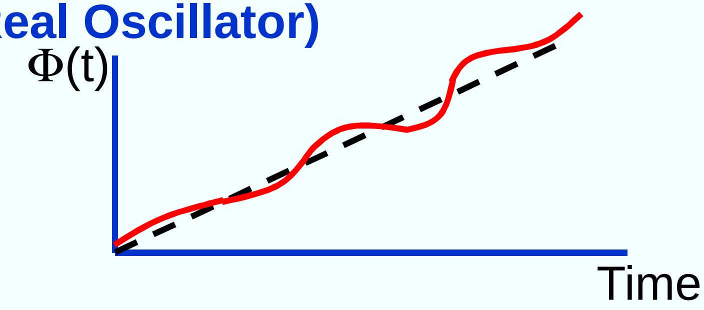


$\Phi(t) = 2\pi\nu_0 t$

Unstable Frequency (Real Oscillator)



$V(t) = [V_0 + \varepsilon(t)] \sin[2\pi\nu_0 t + \phi(t)]$

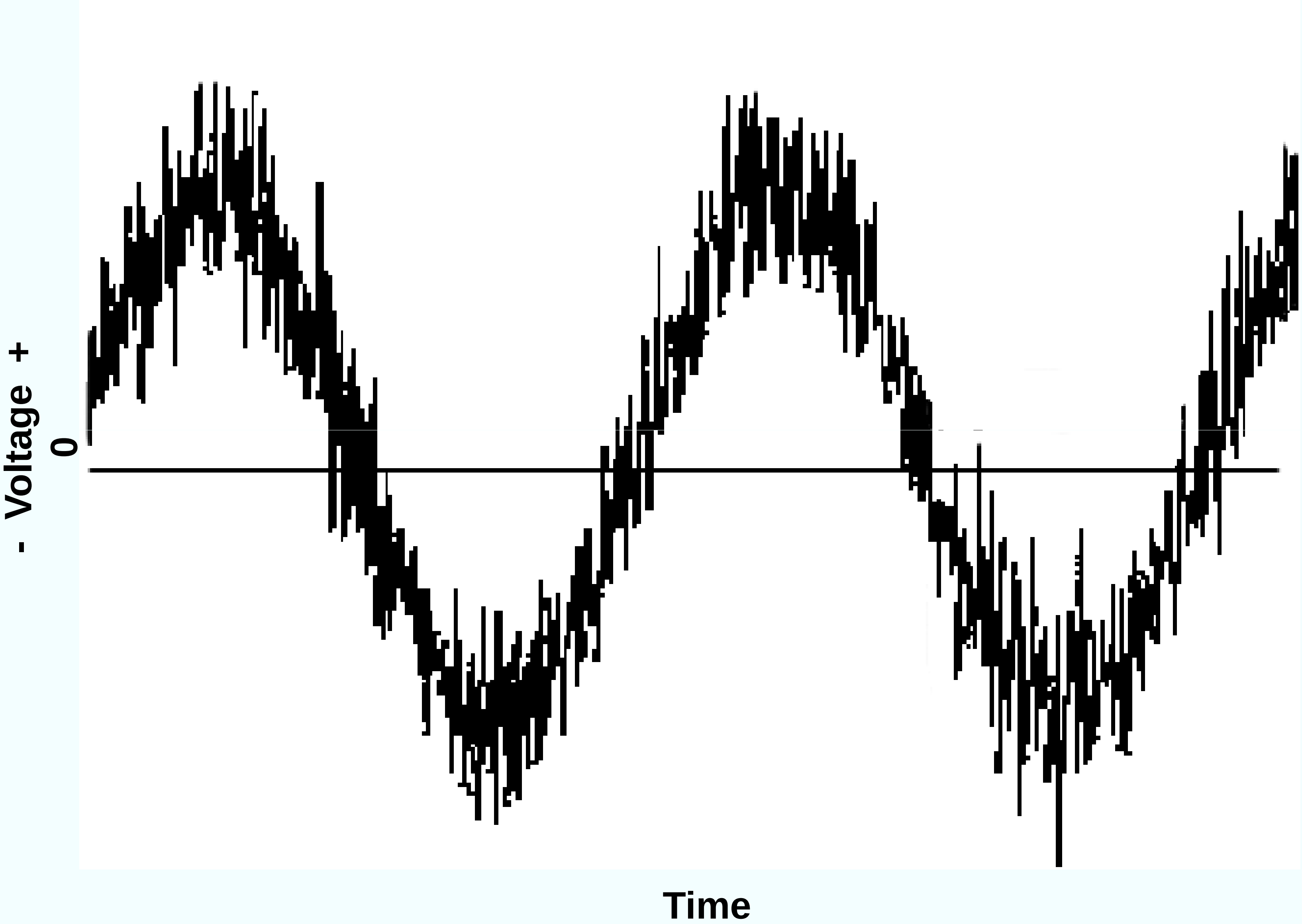


$\Phi(t) = 2\pi\nu_0 t + \phi(t)$

Instantaneous frequency, $\nu(t) = \dfrac{1}{2\pi}\dfrac{d\Phi(t)}{dt} = \nu_0 + \dfrac{1}{2\pi}\dfrac{d\phi(t)}{dt}$

$V(t)$ = Oscillator output voltage, V_0 = Nominal peak voltage amplitude
$\varepsilon(t)$ = Amplitude noise, ν_0 = Nominal (or "carrier") frequency
$\Phi(t)$ = Instantaneous phase, and $\phi(t)$ = Deviation of phase from nominal (i.e., the ideal)

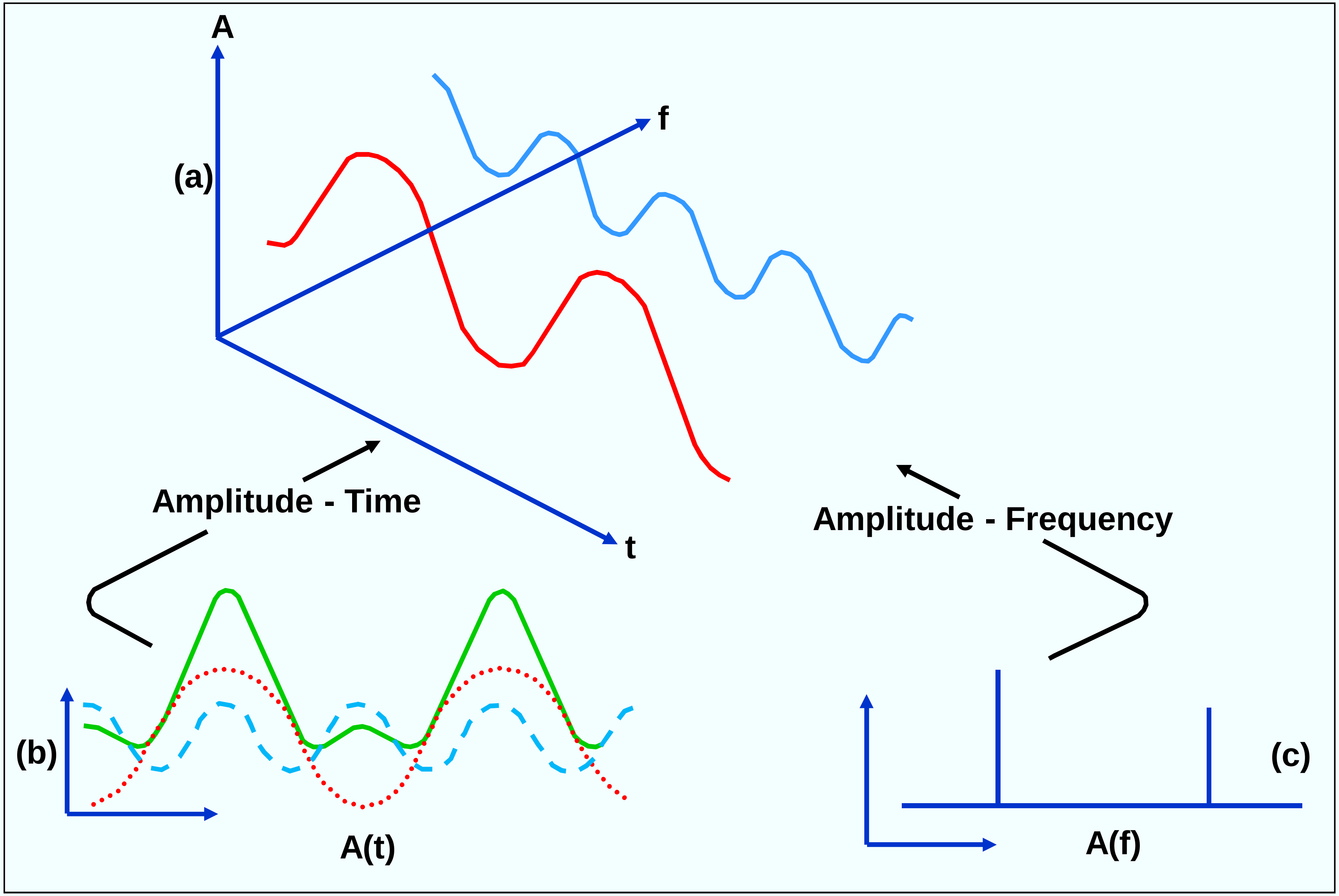
Instantaneous Output Voltage of an Oscillator



Impacts of Oscillator Noise

- Limits the ability to determine the current state and the predictability of oscillators
- Limits syntonization and synchronization accuracy
- Limits receivers' useful dynamic range, channel spacing, and selectivity; can limit jamming resistance
- Limits radar performance (especially Doppler radar's)
- Causes timing errors [~$\tau\sigma_y(\tau)$]
- Causes bit errors in digital communication systems
- Limits number of communication system users, as noise from transmitters interfere with receivers in nearby channels
- Limits navigation accuracy
- Limits ability to lock to narrow-linewidth resonances
- Can cause loss of lock; can limit acquisition/reacquisition capability in phase-locked-loop systems

Time Domain - Frequency Domain



Causes of Short Term Instabilities

- Johnson noise (thermally induced charge fluctuations, i.e., "thermal emf" in resistive elements)
- Phonon scattering by defects & quantum fluctuations (related to Q)
- Noise due to oscillator circuitry (active and passive components)
- Temperature fluctuations - thermal transient effects
  - activity dips at oven set-point
- Random vibration
- Fluctuations in the number of adsorbed molecules
- Stress relief, fluctuations at interfaces (quartz, electrode, mount, bond)
- Shot noise in atomic frequency standards
- ? ? ?

Allan Deviation

Also called **two-sample deviation,** or square-root of the "**Allan variance,**" it is the standard method of describing the short term stability of oscillators in the time domain. It is denoted by $\sigma_y(\tau)$,

where $$\sigma_y^2(\tau) = \frac{1}{2} < (y_{k+1} - y_k)^2 > .$$

The fractional frequencies, $y = \frac{\Delta f}{f}$ are measured over a time interval, τ; $(y_{k+1} - y_k)$ are the differences between pairs of successive measurements of y, and, ideally, < > denotes a time average of an infinite number of $(y_{k+1} - y_k)^2$. A good estimate can be obtained by a limited number, m, of measurements (m≥100). $\sigma_y(\tau)$ generally denotes $\sqrt{\sigma_y^2(\tau, m)}$, i.e.,

$$\sigma_y^2(\tau) = \sigma_y^2(\tau, m) = \frac{1}{m} \sum_{j=1}^{m} \tfrac{1}{2}\left(y_{k+1} - y_k\right)_j^2$$

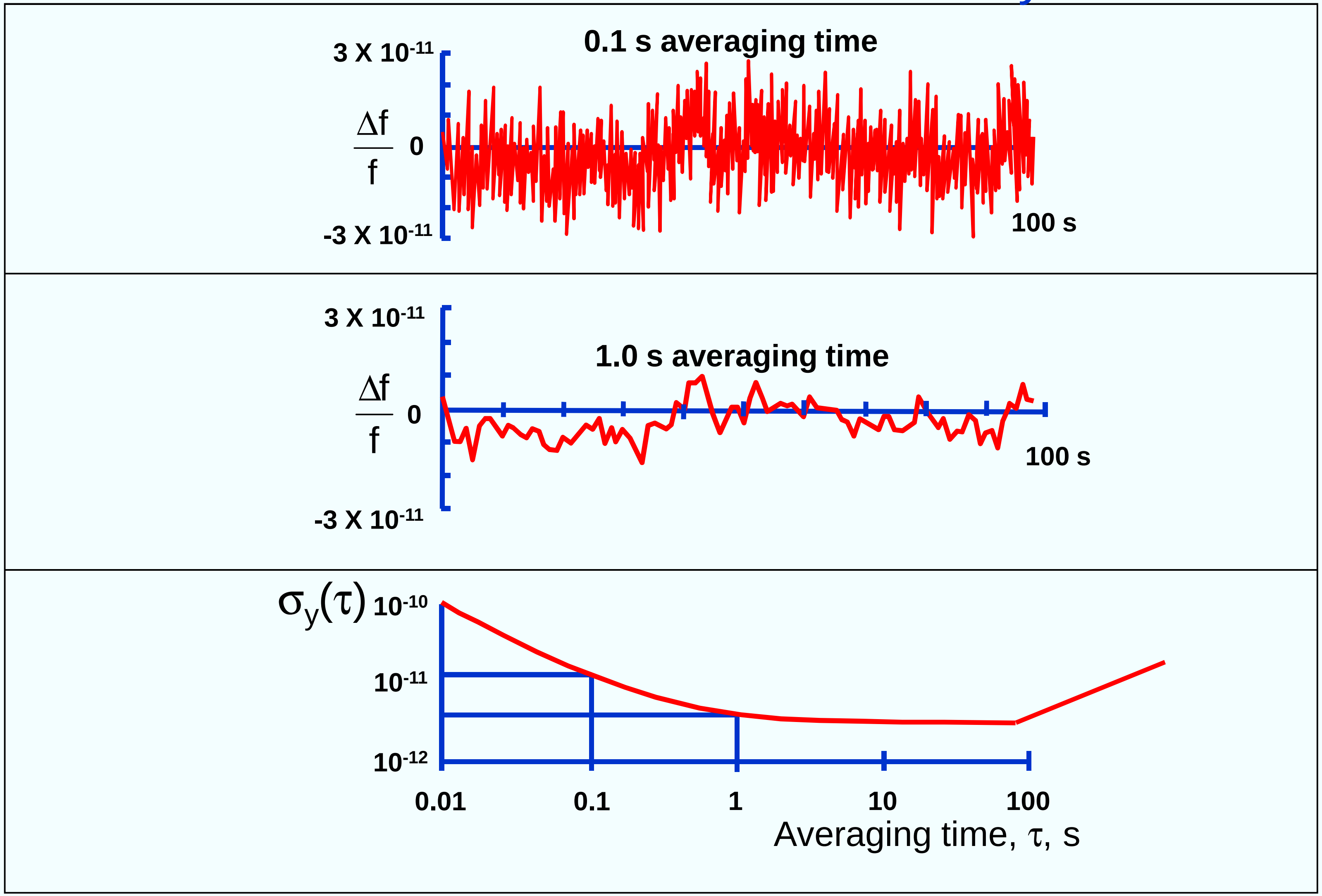
Why $\sigma_y(\tau)$?

- **Classical variance:** $\sigma^2_{y_i} = \frac{1}{m-1}\sum\left(y_i - \overline{y}\right)^2,$

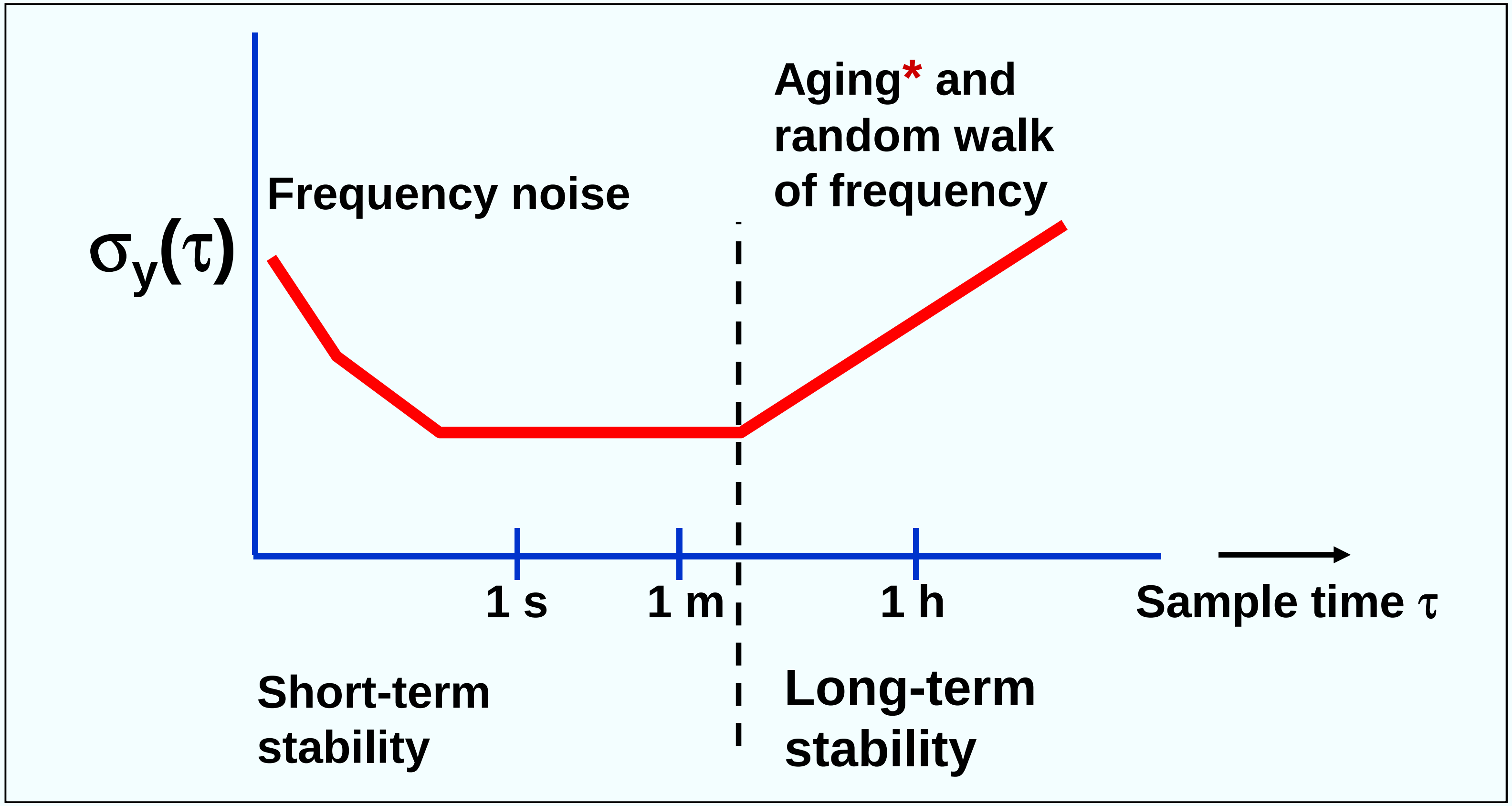
  diverges for some commonly observed noise processes, such as random walk, i.e., the variance increases with increasing number of data points.

- **Allan variance:**
  - Converges for all noise processes observed in precision oscillators.
  - Has straightforward relationship to power law spectral density types.
  - Is easy to compute.
  - Is faster and more accurate in estimating noise processes than the Fast Fourier Transform.

Frequency Noise and $\sigma_y(\tau)$

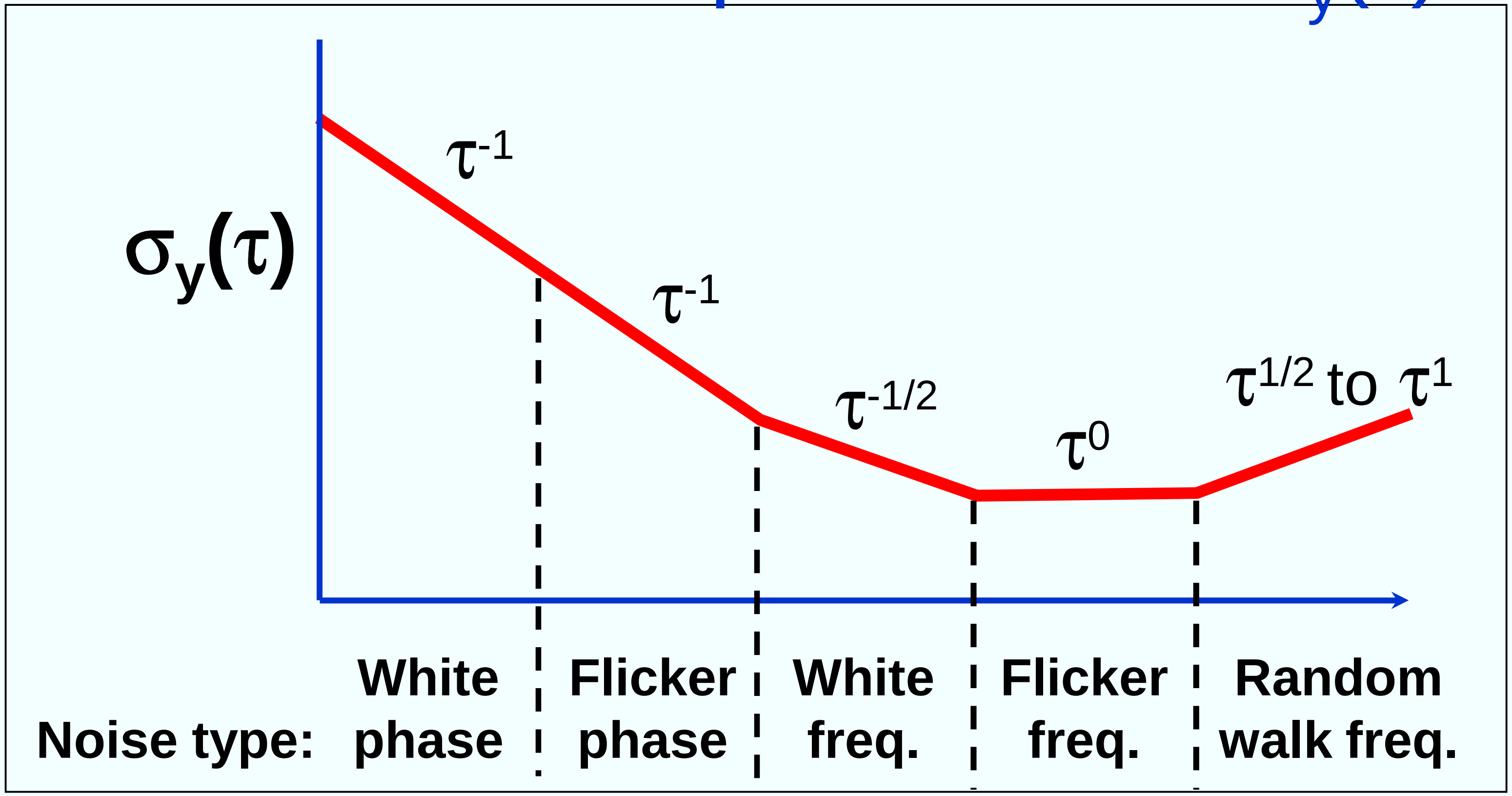


Time Domain Stability



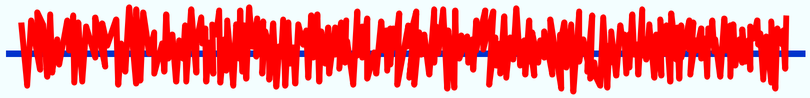
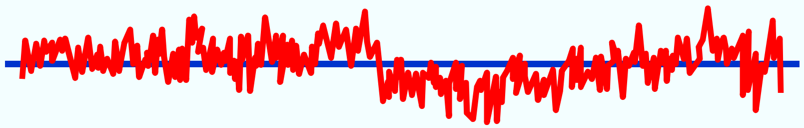
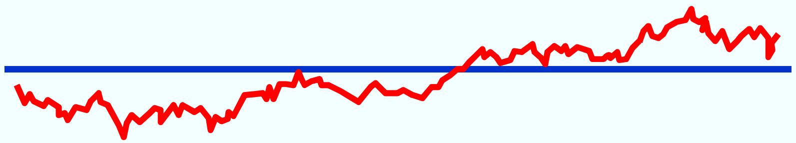
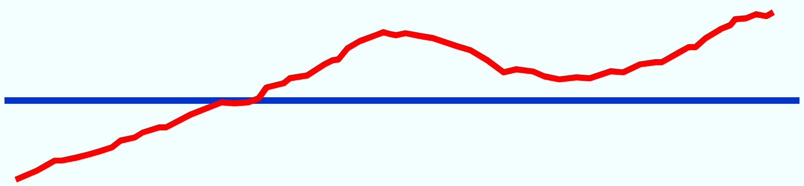
*For $\sigma_y(\tau)$ to be a proper measure of random frequency fluctuations, aging must be properly subtracted from the data at long τ's.

Power Law Dependence of $\sigma_y(\tau)$



Below the flicker of frequency noise (i.e., the "flicker floor") region, crystal oscillators typically show τ^{-1} (white phase noise) dependence. Atomic standards show $\tau^{-1/2}$ (white frequency noise) dependence down to about the servo-loop time constant, and τ^{-1} dependence at less than that time constant. Typical τ's at the start of flicker floors are: 1s for a crystal oscillator, 10^3s for a Rb standard and 10^5s for a Cs standard. At large τ's, random walk of frequency and aging dominate.

Pictures of Noise

Plot of $z(t)$ vs. t	$S_z(f) = h_\alpha f^\alpha$	Noise name
	$\alpha = 0$	White
	$\alpha = -1$	Flicker
	$\alpha = -2$	Random walk
	$\alpha = -3$	

Plots show fluctuations of a quantity $z(t)$, which can be,e.g., the output of a counter (Δf vs. t) or of a phase detector ($\phi[t]$ vs. t). The plots show simulated time-domain behaviors corresponding to the most common (power-law) spectral densities; h_α is an amplitude coefficient. Note: since $S_{\Delta f} = f^{\,2} S_\phi$, e.g. white frequency noise and random walk of phase are equivalent.

Spectral Densities

$$V(t) = \left[V_0 + \varepsilon(t)\right] \sin\left[2\pi\nu_0 t + \phi(t)\right]$$

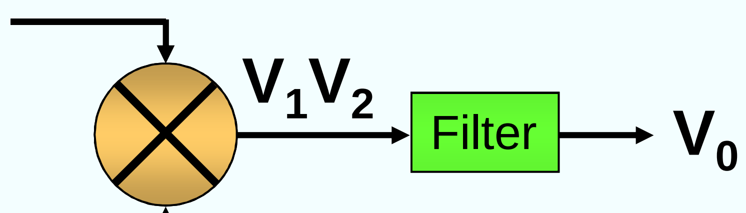
In the frequency domain, due to the phase deviation, $\phi(t)$, some of the power is at frequencies other than ν_0. The stabilities are characterized by "spectral densities." The spectral density, $S_V(f)$, the mean-square voltage $<V^2(t)>$ in a unit bandwidth centered at f, is not a good measure of frequency stability because both $\varepsilon(t)$ and $\phi(t)$ contribute to it, and because it is not uniquely related to frequency fluctuations (although $\varepsilon(t)$ is often negligible in precision frequency sources.)

The spectral densities of phase and fractional-frequency fluctuations, $S_\phi(f)$ and $S_y(f)$, respectively, are used to measure the stabilities in the frequency domain. The spectral density $S_g(f)$ of a quantity g(t) is the mean square value of g(t) in a unit bandwidth centered at f. Moreover, the RMS value of g^2 in bandwidth BW is given by $g^2_{RMS}(t) = \int_{BW} S_g(f)df$.

Mixer Functions

$$V_1 = A_1 \sin(\omega_1 t + \varphi_1)$$

$$V_2 = A_2 \sin(\omega_2 t + \varphi_2)$$



Trigonometric identities: sin(x)sin(y) = ½cos(x-y) - ½cos(x+y)

$\cos(x \pm \pi/2) = \sin(x)$

Let $\omega_1 = \omega_2; \Phi_1 \equiv \omega_1 t + \varphi_1$, and $\Phi_2 \equiv \omega_2 t + \varphi_2$. Then the mixer can become :

- **Phase detector:** When $\Phi_1 = \Phi_2 + \pi/2$ and $A_1 = A_2 = 1$, then

$$V_0 = \frac{1}{2}\sin(\varphi_1 - \varphi_2) = \frac{1}{2}(\varphi_1 - \varphi_2) \text{ for small } \varphi\text{'s}$$

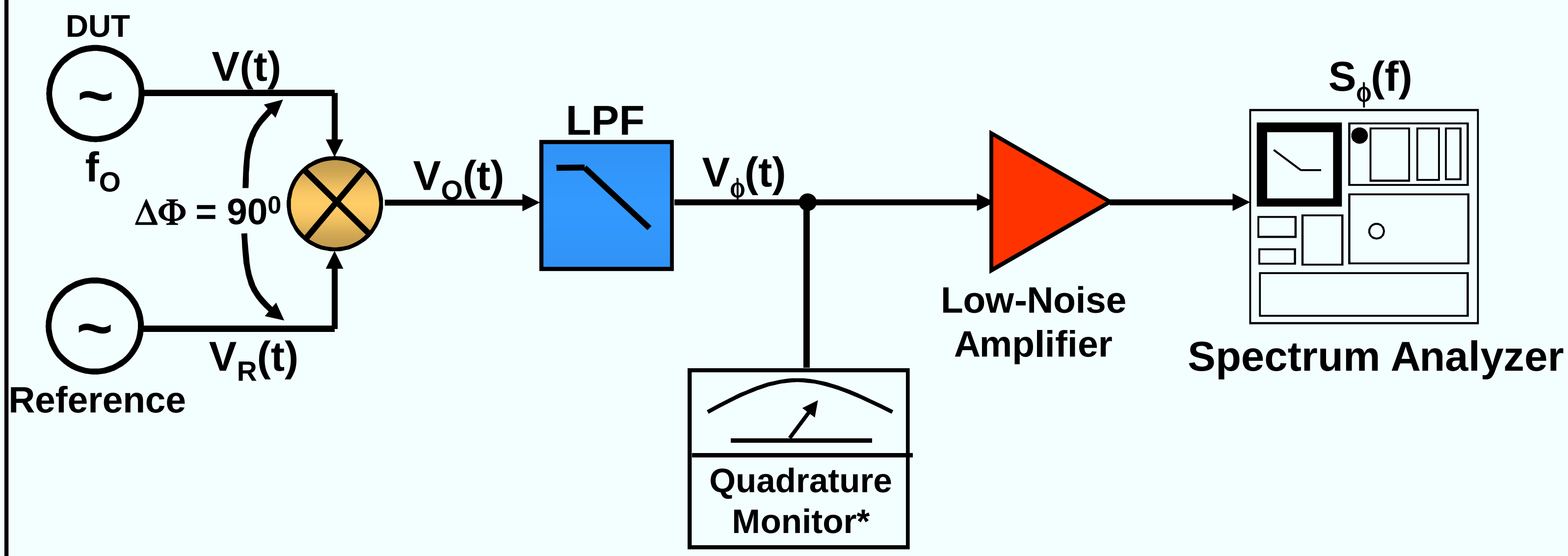
- **AM detector:** When $A_2 = 1$ and the filter is a low-pass filter, then

$$V_0 = \frac{1}{2}A_1\cos(\varphi_1 - \varphi_2); \text{ if } \varphi_1 \approx \varphi_2, \text{ then } V_0 \approx \frac{1}{2}A_1$$

- **Frequency multiplier:** When $V_1 = V_2$ and the filter is bandpass at $2\omega_1$

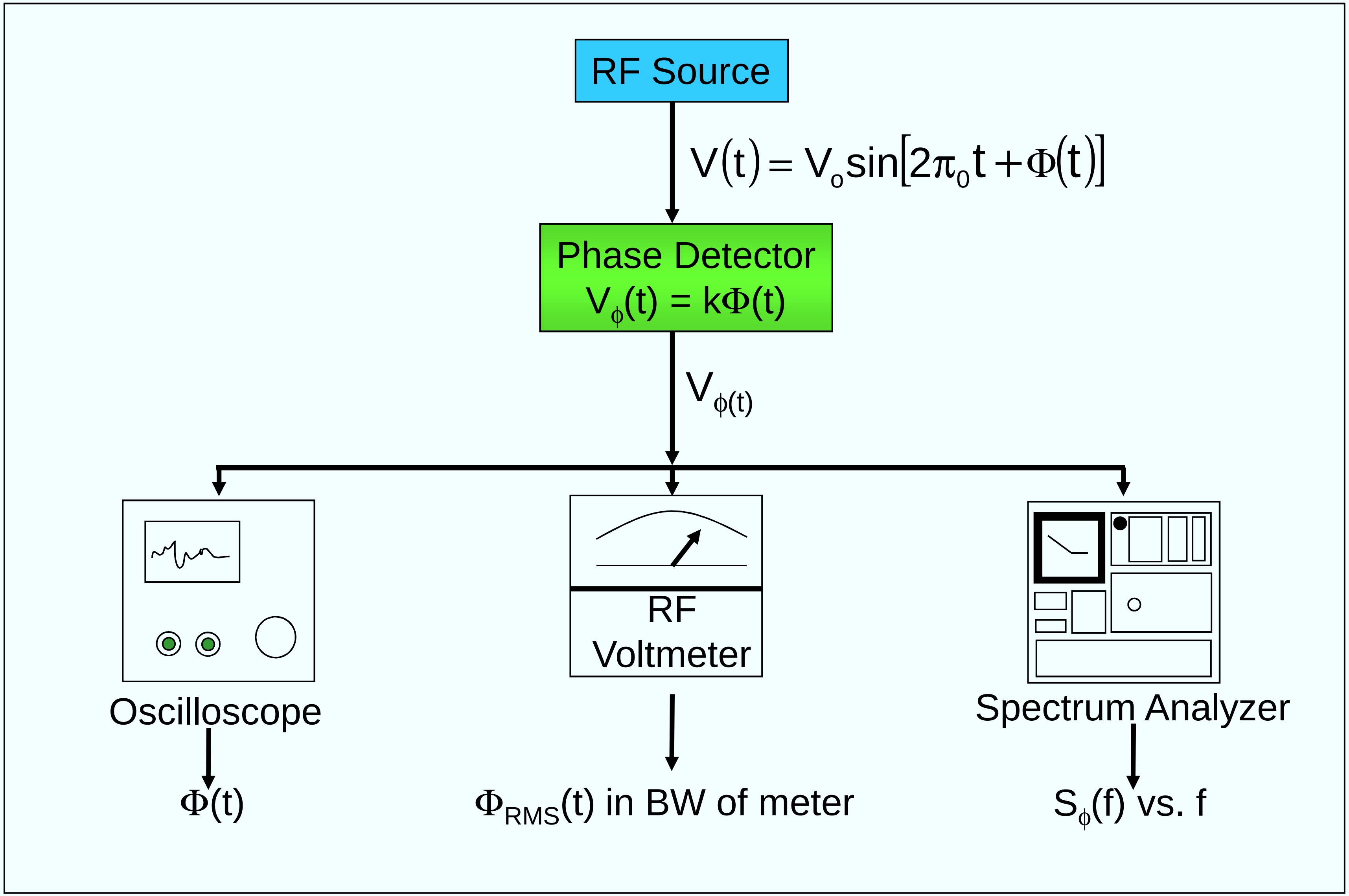
then, $V_0 = \frac{1}{2}A_1^2\cos(2\omega_1 t + 2\varphi_1) \Rightarrow$ doubles the frequency and phase error.

Phase Detector



* Or phase-locked loop

Phase Noise Measurement



Frequency - Phase - Time Relationships

$$\nu(t) = \nu_0 + \frac{1}{2\pi}\frac{d\phi(t)}{dt} = \text{"instantaneous" frequency;} \quad \phi(t) = \phi_0 + \int_0^t 2\pi\left[\nu(t') - \nu_0\right]dt'$$

$$y(t) \equiv \frac{\nu(t) - \nu_0}{\nu_0} = \frac{\dot{\phi}(t)}{2\pi\nu_0} = \text{normalized frequency;} \quad \phi_{RMS}^2 = \int S_\phi(f)\,dt$$

$$S_\phi(f) = \frac{\phi_{RMS}^2}{BW} = \left(\frac{\nu_0}{f}\right)^2 S_y(f); \qquad \mathcal{L}(f) \equiv 1/2\, S_\phi(f), \text{ per IEEE Standard 1139} - 1988$$

$$\sigma_y^2(\tau) = 1/2 < \left(\bar{y}_{k+1} - \bar{y}_k\right)^2 > = \frac{2}{\left(\pi\nu_0\tau\right)^2}\int_0^\infty S_\phi(f)\sin^4(\pi f\tau)\,df$$

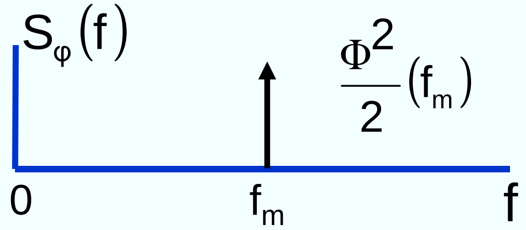
The five common power-law noise processes in precision oscillators are:

$$S_y(f) = \underset{\text{(White PM)}}{h_2 f^2} + \underset{\text{(Flicker PM)}}{h_1 f} + \underset{\text{(White FM)}}{h_0} + \underset{\text{(Flicker FM)}}{h_{-1} f^{-1}} + \underset{\text{(Random-walk FM)}}{h_{-2} f^{-2}}$$

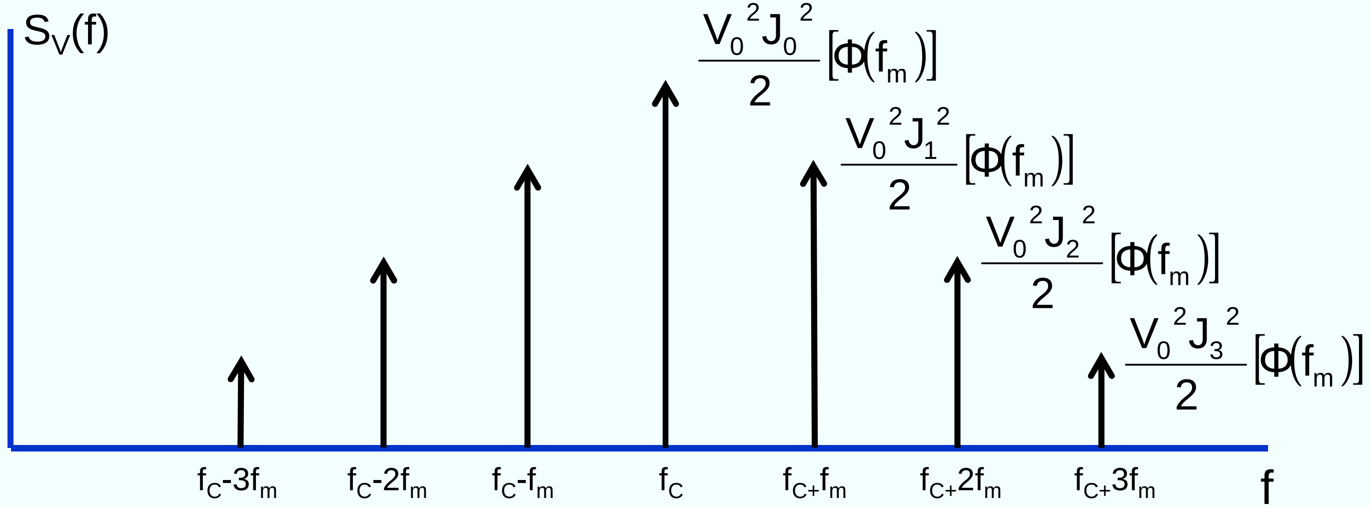
$$\text{Time deviation} = x(t) = \int_o^t y(t')\,dt' = \frac{\phi(t)}{2\pi\nu}$$

$S_\phi(f)$, $S_V(f)$ and $\mathcal{L}(f)$

$$\Phi(t) = \Phi(f_m)\cos(2\pi f_m t)$$

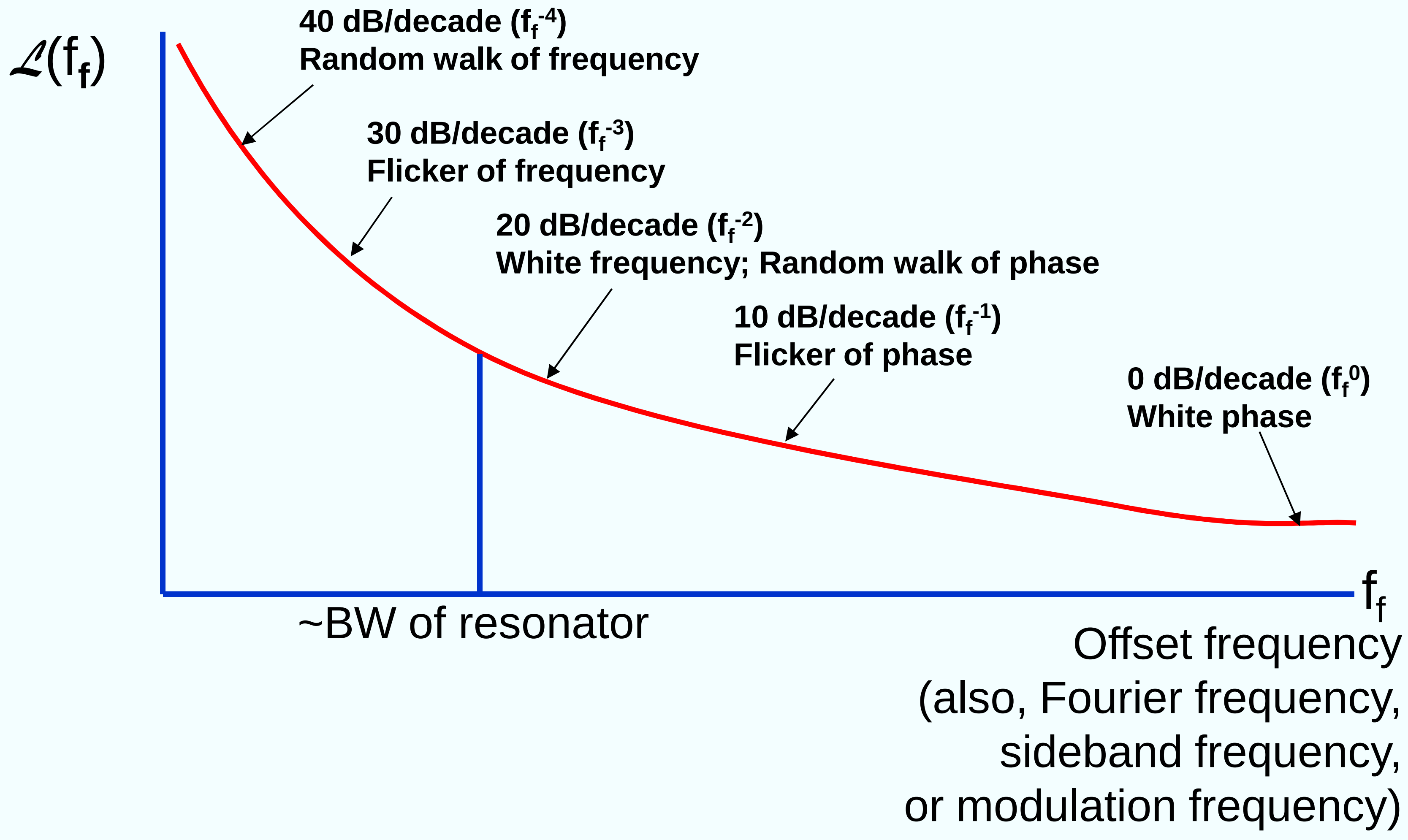


$$V(t) = V_0 \cos\left[2\pi f_C t + \Phi(f_m)\right]$$



$$\text{SSB Power Ratio} = \frac{V_0^{\,2} J_1^{\,2}\left[\Phi(f_m)\right]}{V_0^{\,2} J_0^{\,2}\left[\Phi(f_m)\right] + 2\sum_{i=1}^{\infty} J_i^{2}\left[\Phi\left(f_m\right)\right]} \cong \mathcal{L}(f_m) \equiv \frac{S\varphi(f_m)}{2}$$

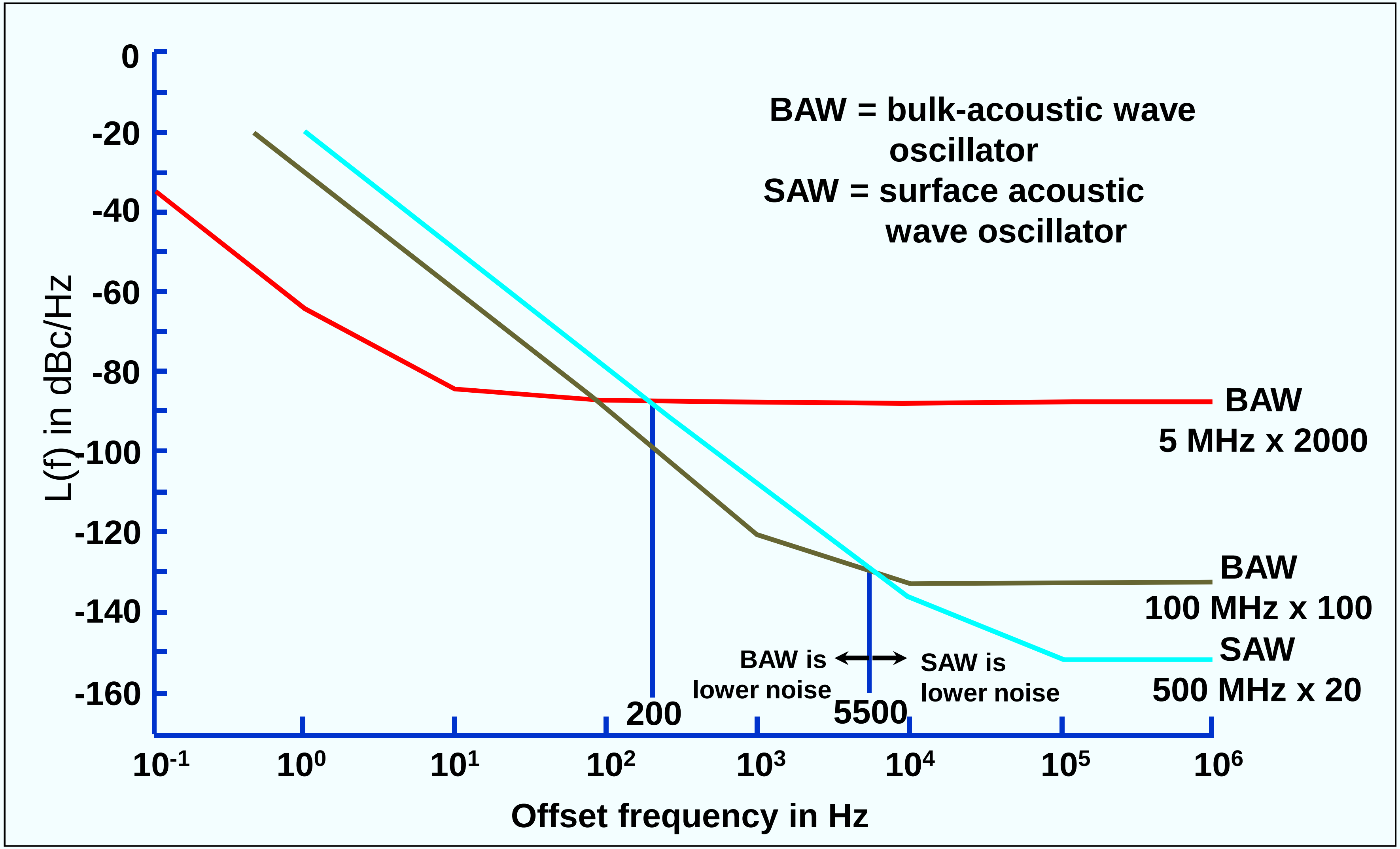
Types of Phase Noise



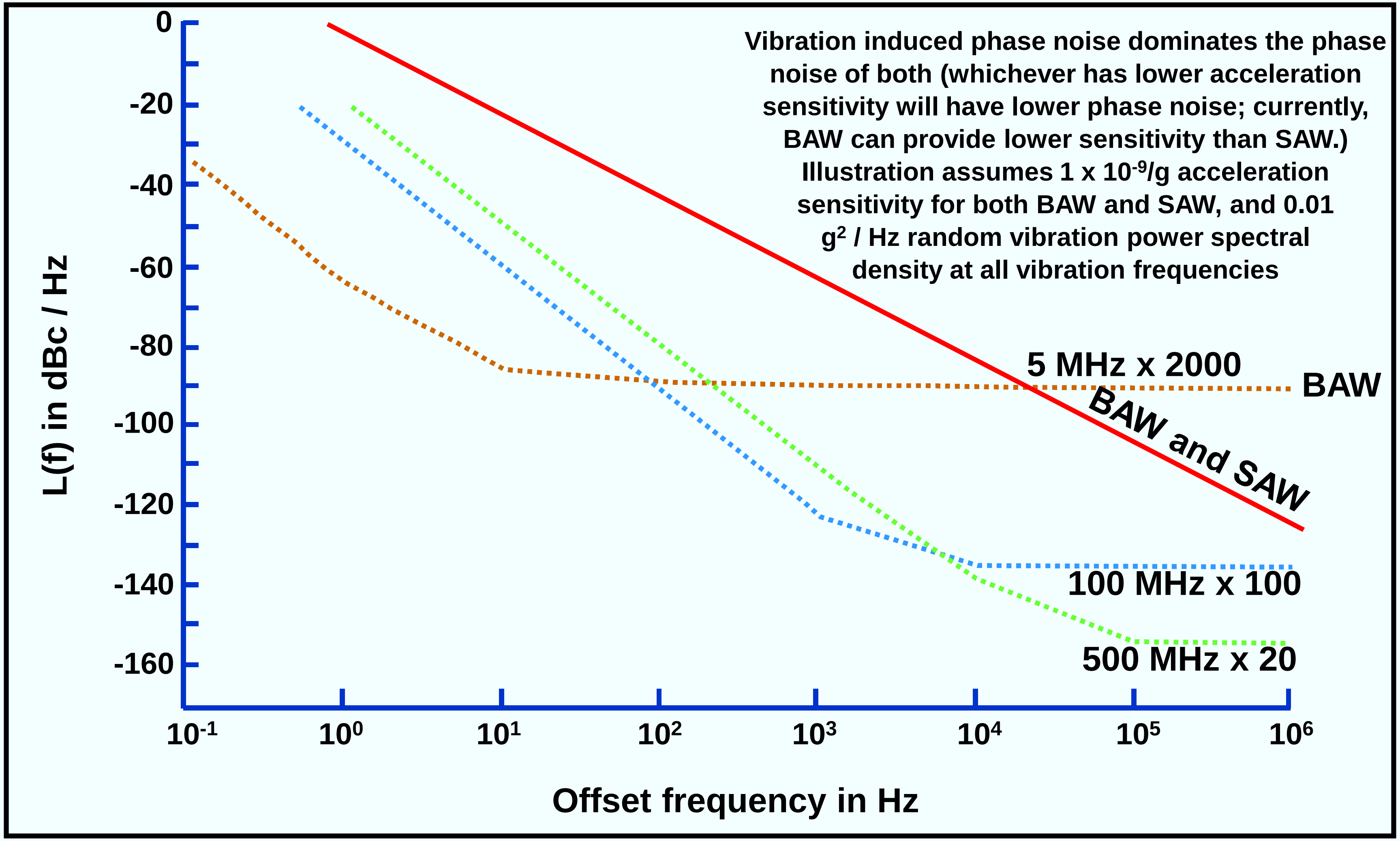
Noise in Crystal Oscillators

- The resonator is the primary noise source close to the carrier; the oscillator sustaining circuitry is the primary source far from the carrier.
- Frequency multiplication by N increases the phase noise by N^2 (i.e., by 20log N, in dB's).
- Vibration-induced "noise" dominates all other sources of noise in many applications (see acceleration effects section, later).
- Close to the carrier (within BW of resonator), $S_y(f)$ varies as 1/f, $S_\phi(f)$ as $1/f^3$, where f = offset from carrier frequency, ν. $S_\phi(f)$ also varies as $1/Q^4$, where Q = unloaded Q. Since $Q_{max}\nu$ = const., $S_\phi(f) \propto \nu^4$. $(Q_{max}\nu)_{BAW} = 1.6 \times 10^{13}$ Hz; $(Q_{max}\nu)_{SAW} = 1.05 \times 10^{13}$ Hz.
- In the time domain, noise floor is $\sigma_y(\tau) \geq (2.0 \times 10^{-7})Q^{-1} \approx 1.2 \times 10^{-20}\nu$, ν in Hz. In the regions where $\sigma_y(\tau)$ varies as τ^{-1} and $\tau^{-1/2}$ ($\tau^{-1/2}$ occurs in atomic frequency standards), $\sigma_y(\tau) \propto (QS_R)^{-1}$, where S_R is the signal-to-noise ratio; i.e., the higher the Q and the signal-to-noise ratio, the better the short term stability (and the phase noise far from the carrier, in the frequency domain).
- It is the loaded Q of the resonator that affects the noise when the oscillator sustaining circuitry is a significant noise source.
- Noise floor is limited by Johnson noise; noise power, kT = -174 dBm/Hz at 290°K.
- Resonator amplitude-frequency effect can contribute to amplitude and phase noise.
- Higher signal level improves the noise floor but not the close-in noise. (In fact, high drive levels generally degrade the close-in noise, for reasons that are not fully understood.)
- Low noise SAW vs. low noise BAW multiplied up: BAW is lower noise at f < ~1 kHz, SAW is lower noise at f > ~1 kHz; can phase lock the two to get the best of both.

Low-Noise SAW and BAW Multiplied to 10 GHz
(in a nonvibrating environment)



Low-Noise SAW and BAW Multiplied to 10 GHz (in a vibrating environment)



TCXO Noise

The short term stabilities of TCXOs are temperature (T) dependent, and are generally worse than those of OCXOs, for the following reasons:

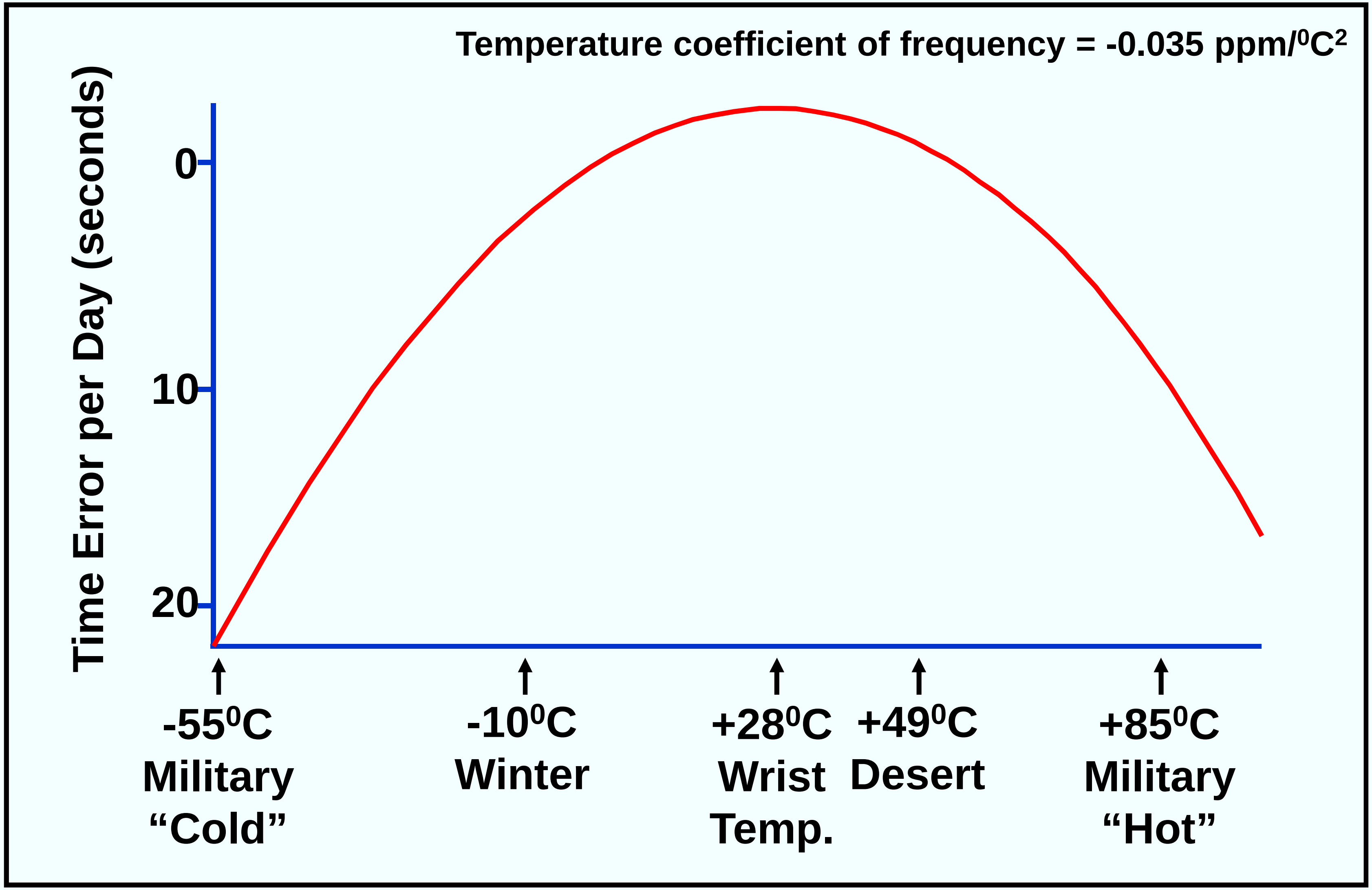
- The slope of the TCXO crystal's frequency (f) vs. T varies with T. For example, the f vs. T slope may be near zero at ~20^{o}C, but it will be ~1ppm/^{o}C at the T extremes. T fluctuations will cause small f fluctuations at laboratory ambient T's, so the stability can be good there, but millidegree fluctuations will cause ~10^{-9} f fluctuations at the T extremes. The TCXO's f vs. T slopes also vary with T; the zeros and maxima can be at any T, and the maximum slopes can be on the order of 1 ppm/^{o}C.

- AT-cut crystals' thermal transient sensitivity makes the effects of T fluctuations depend not only on the T but also on the rate of change of T (whereas the SC-cut crystals typically used in precision OCXOs are insensitive to thermal transients). Under changing T conditions, the T gradient between the T sensor (thermistor) and the crystal will aggravate the problems.

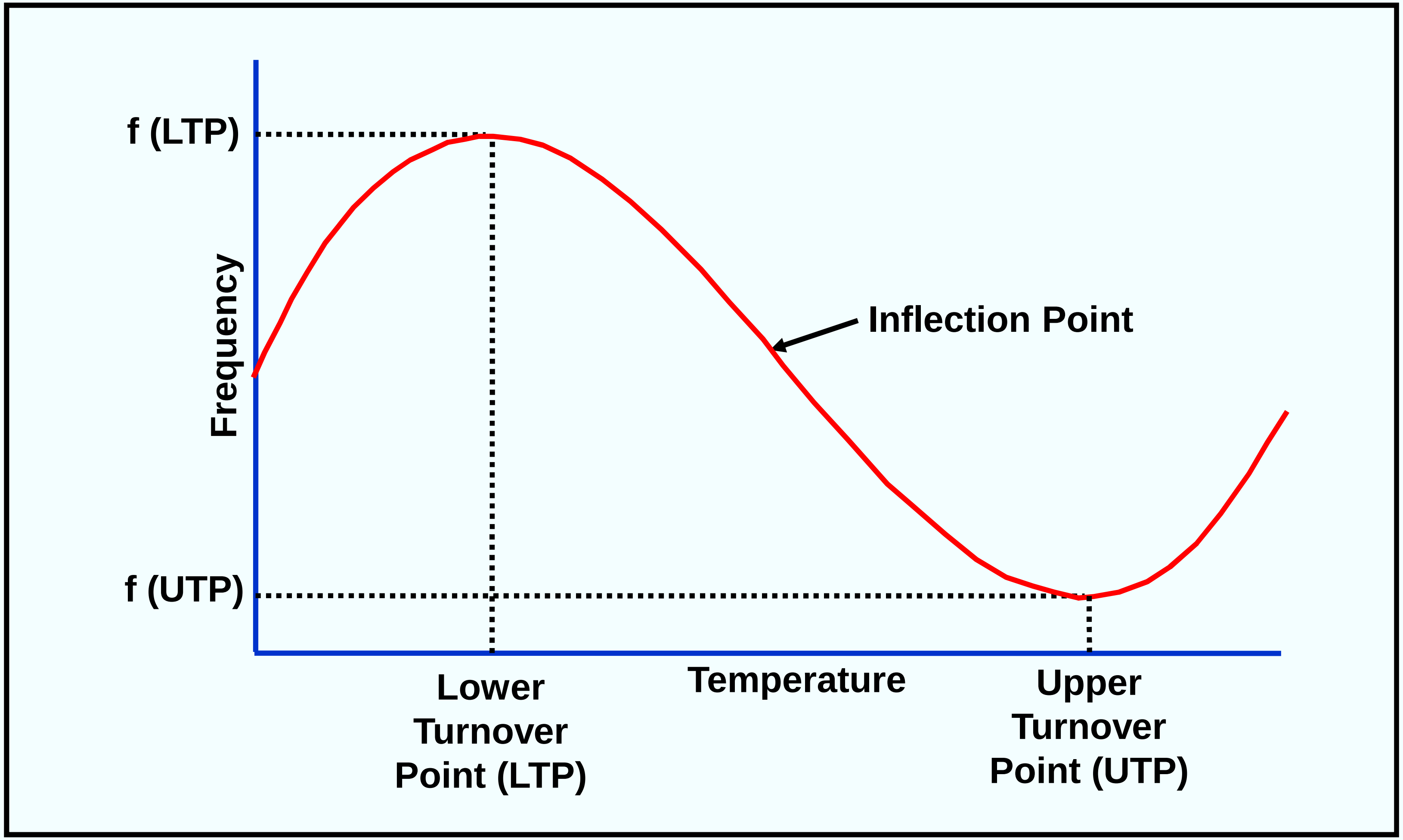
- TCXOs typically use fundamental mode AT-cut crystals which have lower Q and larger C_1 than the crystals typically used in OCXOs. The lower Q makes the crystals inherently noisier, and the larger C_1 makes the oscillators more susceptible to circuitry noise.

- AT-cut crystals' f vs. T often exhibit activity dips (see "Activity Dips" later in this chapter). At the T's where the dips occur, the f vs. T slope can be very high, so the noise due to T fluctuations will also be very high, e.g., 100x degradation of $\sigma_y(\tau)$ and 30 dB degradation of phase noise are possible. Activity dips can occur at any T.

Quartz Wristwatch Accuracy vs. Temperature



Frequency vs. Temperature Characteristics

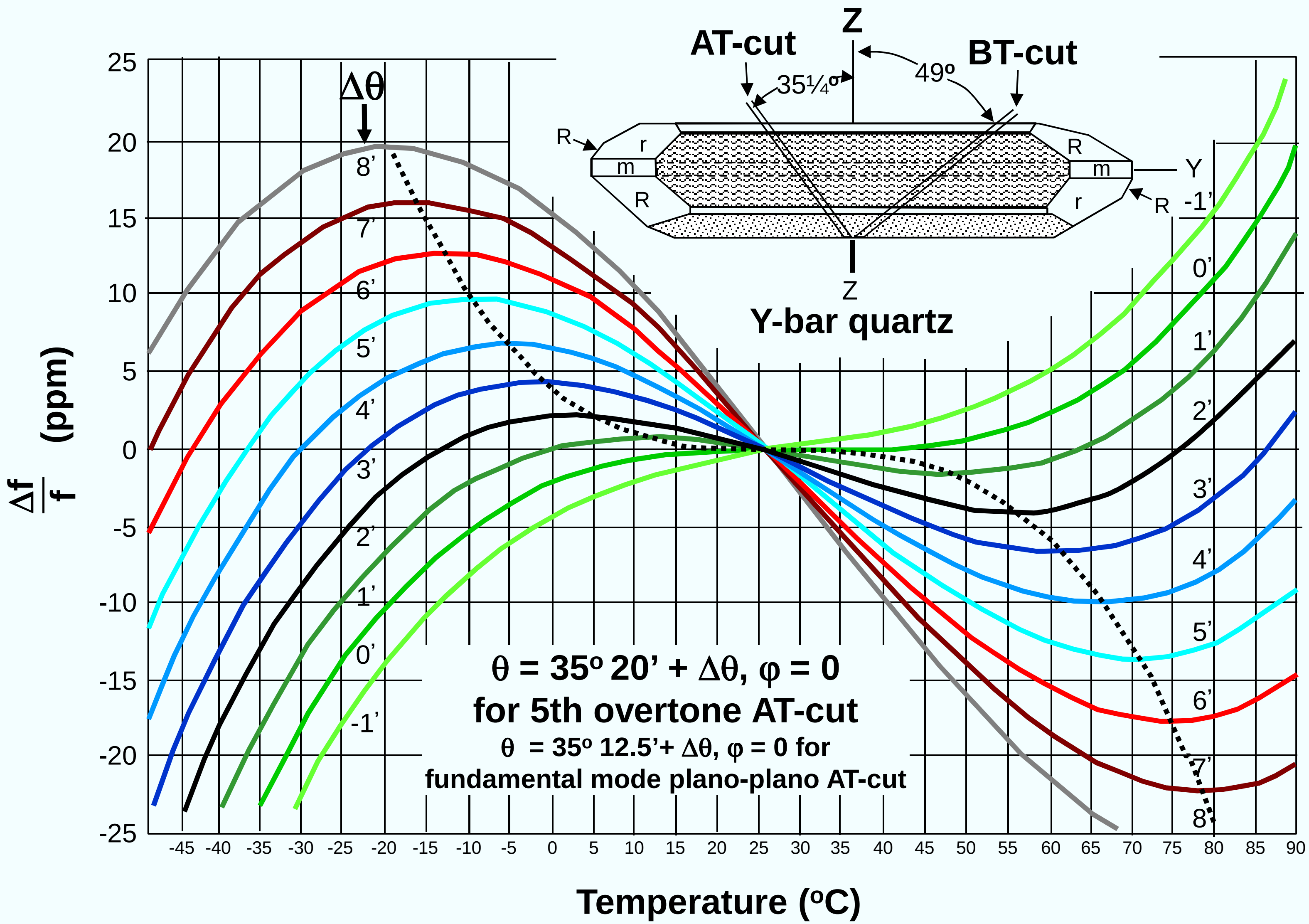


Resonator f vs. T Determining Factors

- **Primary:** Angles of cut

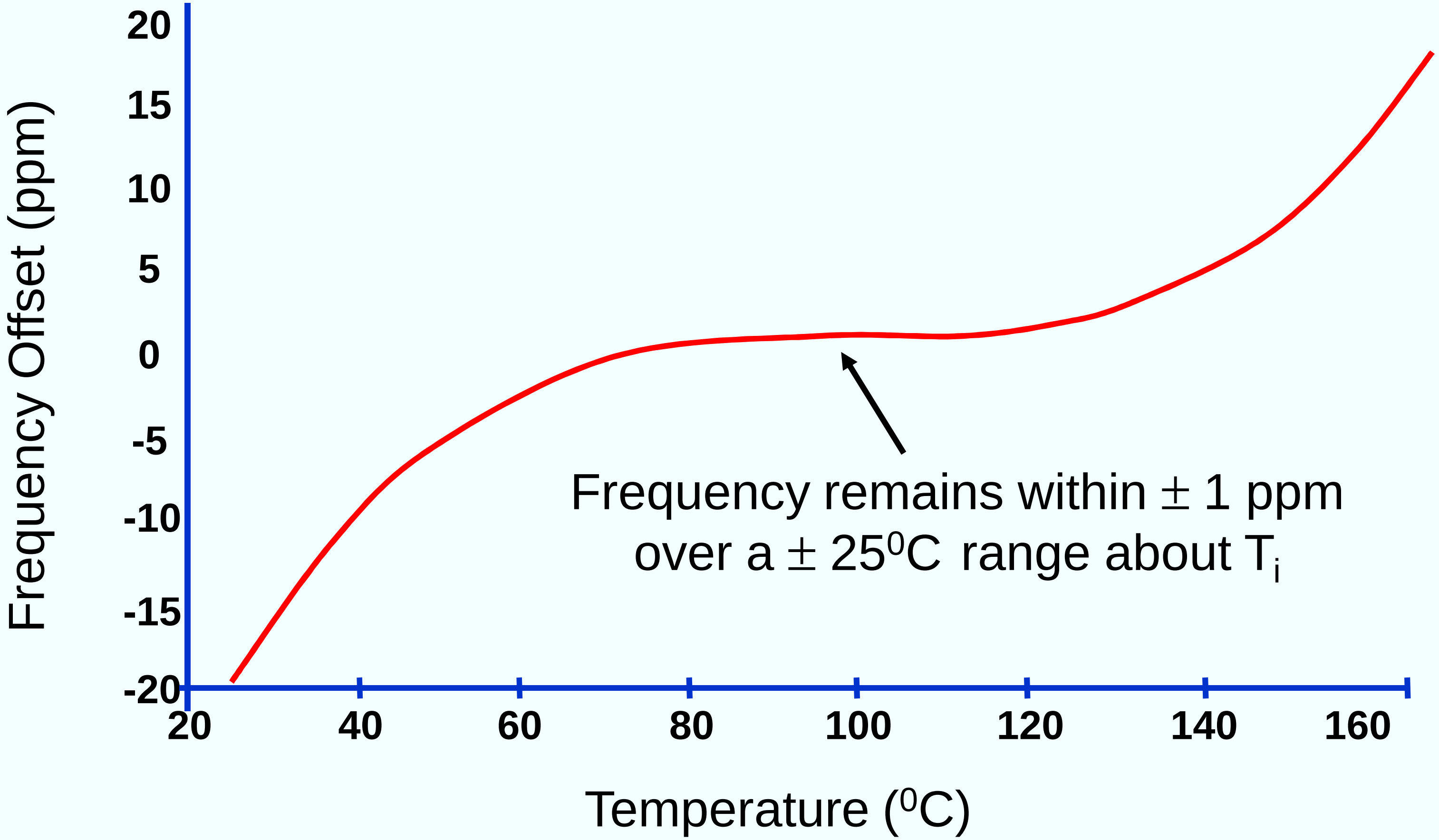
- **Secondary:**
  - Overtone
  - Blank geometry (contour, dimensional ratios)
  - Material impurities and strains
  - Mounting & bonding stresses (magnitude and direction)
  - Electrodes (size, shape, thickness, density, stress)
  - Drive level
  - Interfering modes
  - Load reactance (value & temperature coefficient)
  - Temperature rate of change
  - Thermal history
  - Ionizing radiation

Frequency-Temperature vs. Angle-of-Cut, AT-cut

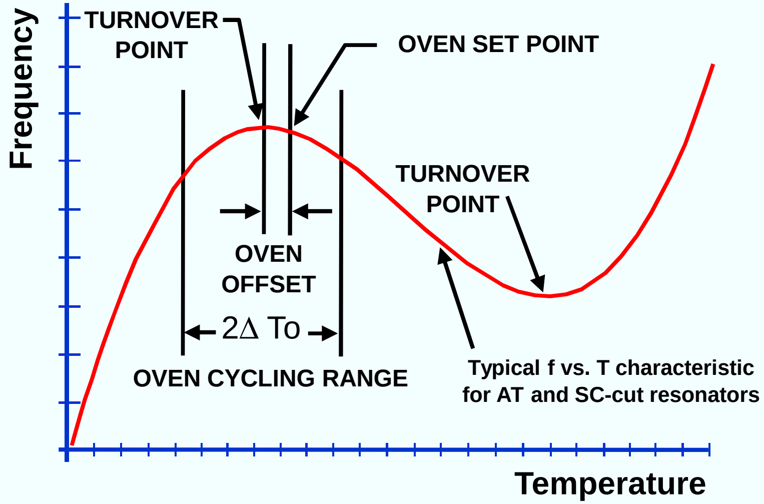


Desired f vs. T of SC-cut Resonator

for OCXO Applications



OCXO Oven's Effect on Stability



Oven Parameters vs. Stability for SC-cut Oscillator
Assuming $T_i - T_{LTP} = 10^0C$

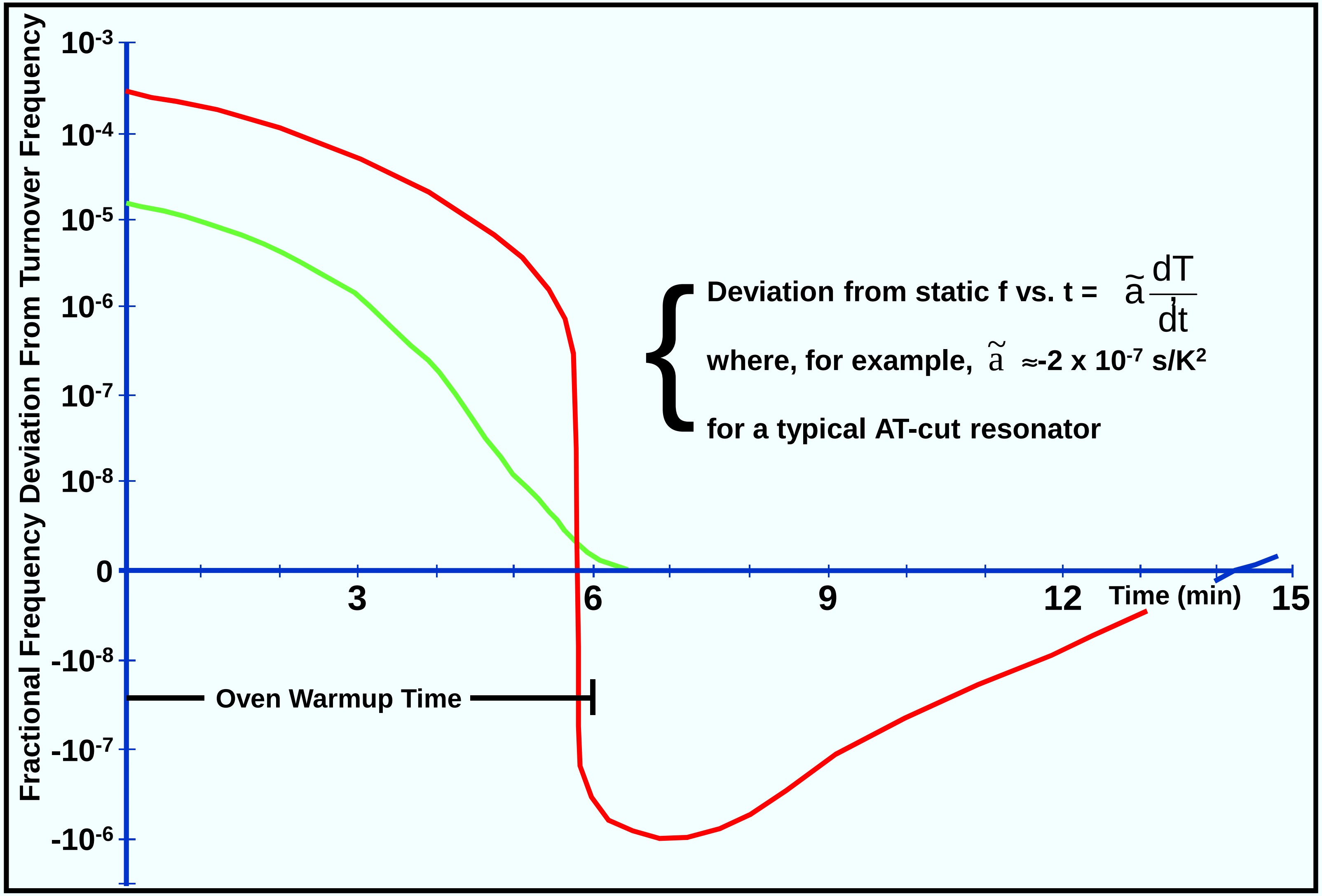
$T_i - T_{LTP} = 10^0C$		Oven Cycling Range (millidegrees)			
		10	1	0.1	0.01
Oven Offset (millidegrees)	100	4×10^{-12}	4×10^{-13}	4×10^{-14}	4×10^{-15}
	10	6×10^{-13}	4×10^{-14}	4×10^{-15}	4×10^{-16}
	1	2×10^{-13}	6×10^{-15}	4×10^{-16}	4×10^{-17}
	0.1	2×10^{-13}	2×10^{-15}	6×10^{-17}	4×10^{-18}
	0	2×10^{-13}	2×10^{-15}	2×10^{-17}	2×10^{-19}

A comparative table for AT and other non-thermal-transient compensated cuts of oscillators would not be meaningful because the dynamic f vs. T effects would generally dominate the static f vs. T effects.

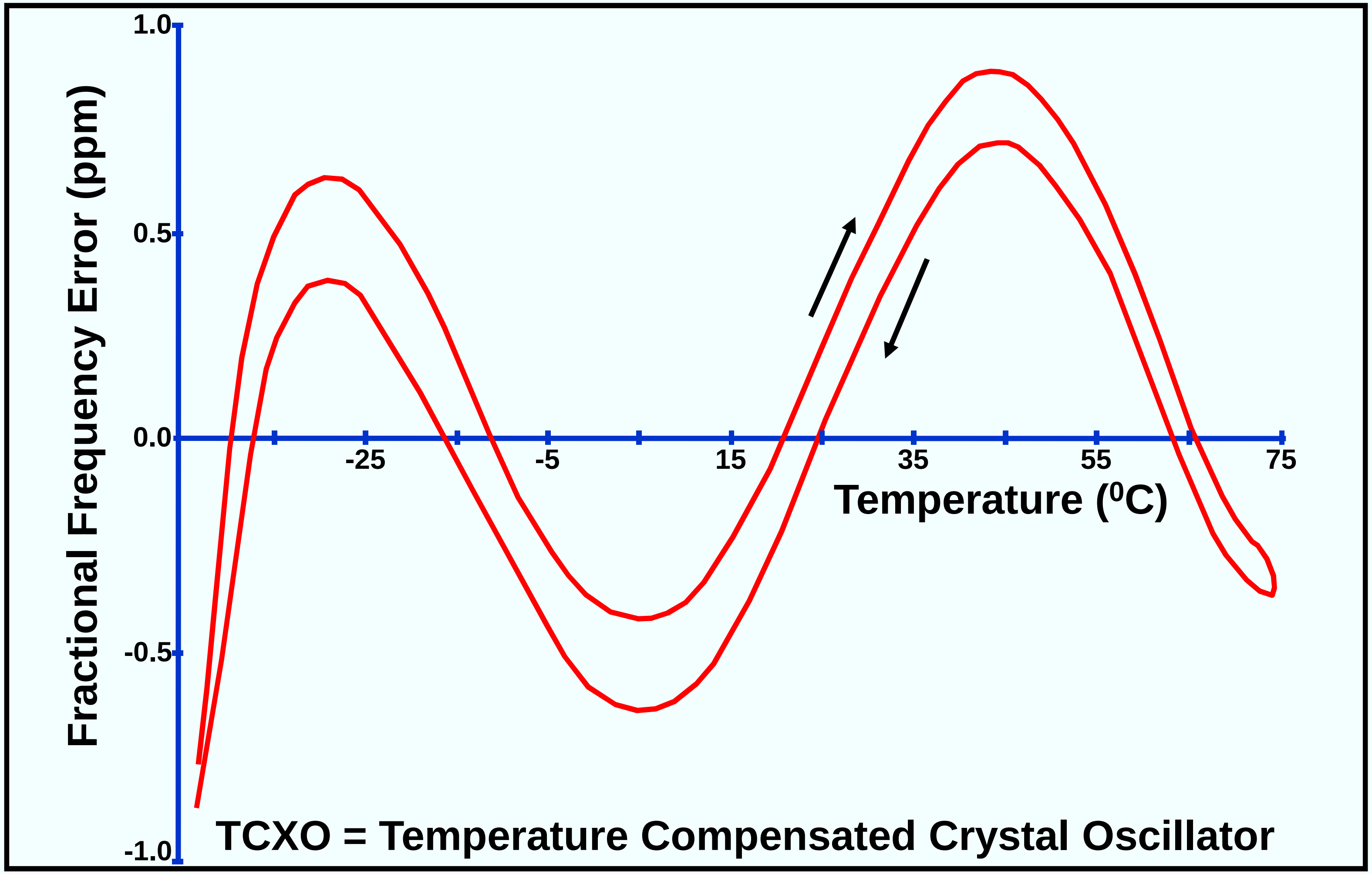
Oven Stability Limits

- Thermal gains of 10^5 has been achieved with a feed-forward compensation technique (i.e., measure outside T of case & adjust setpoint of the thermistor to anticipate and compensate), and with double ovens. For example, with a 10^5 gain, if outside $\Delta T = 100^{o}C$, inside $\Delta T = 1$ mK.
- Stability of a good amplifier ~1μK/K
- Stability of thermistors ~1mK/year to 100mK/year
- Noise < 1μK (Johnson noise in thermistor + amplifier noise + shot noise in the bridge current)
- Quantum limit of temperature fluctuations ~ 1nK
- Optimum oven design can provide very high f vs. T stability

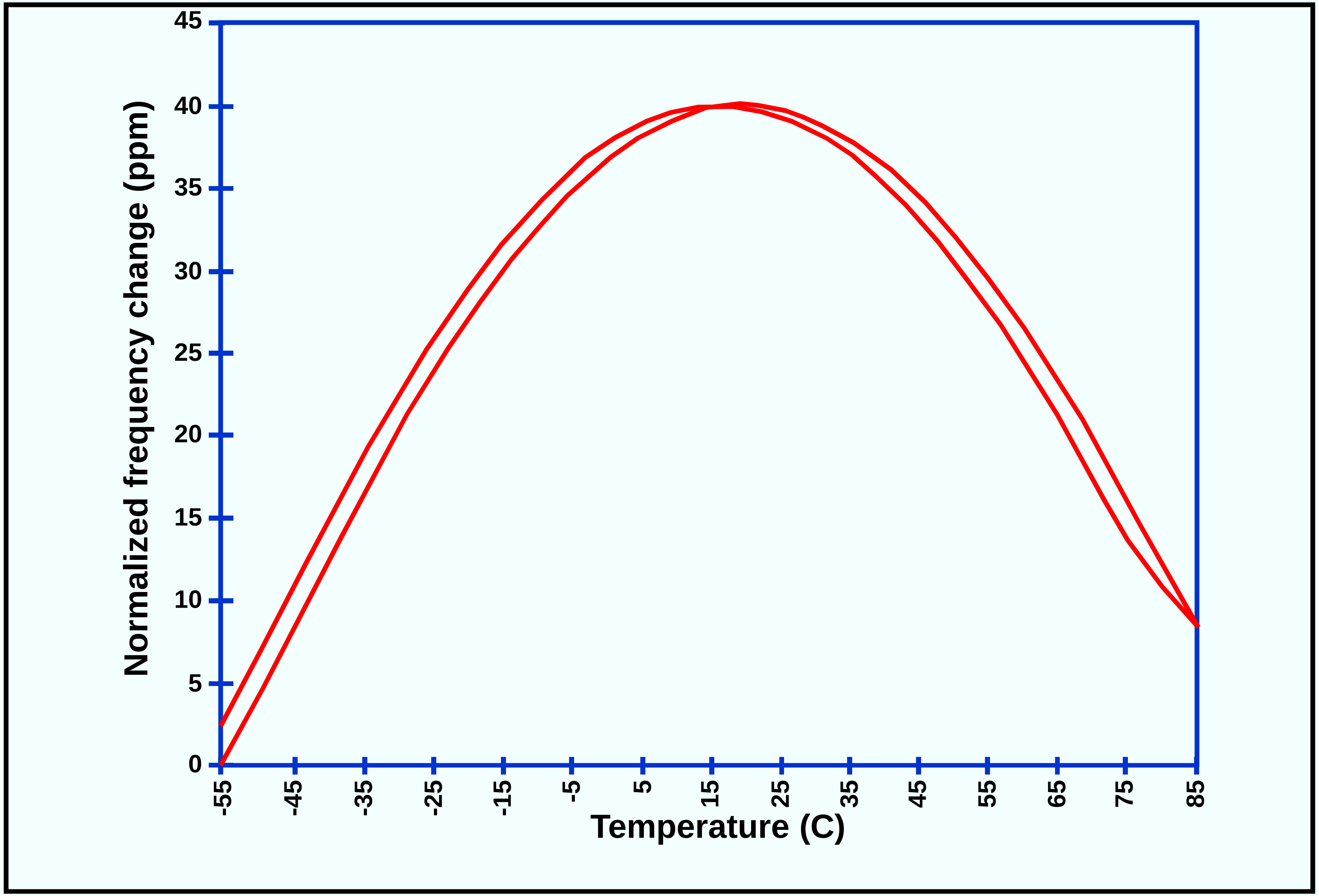
Warmup of AT- and SC-cut Resonators



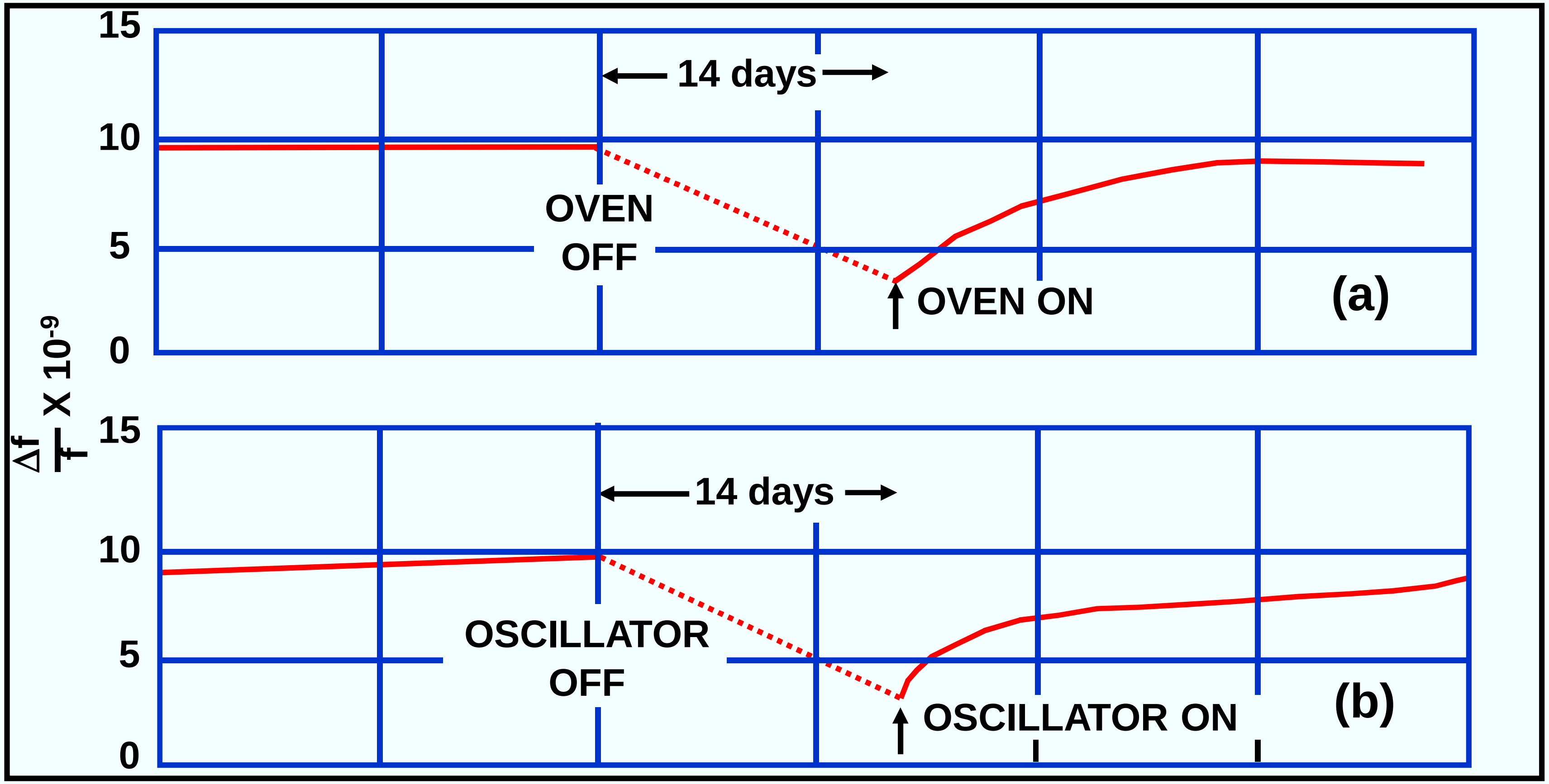
TCXO Thermal Hysteresis



Apparent Hysteresis

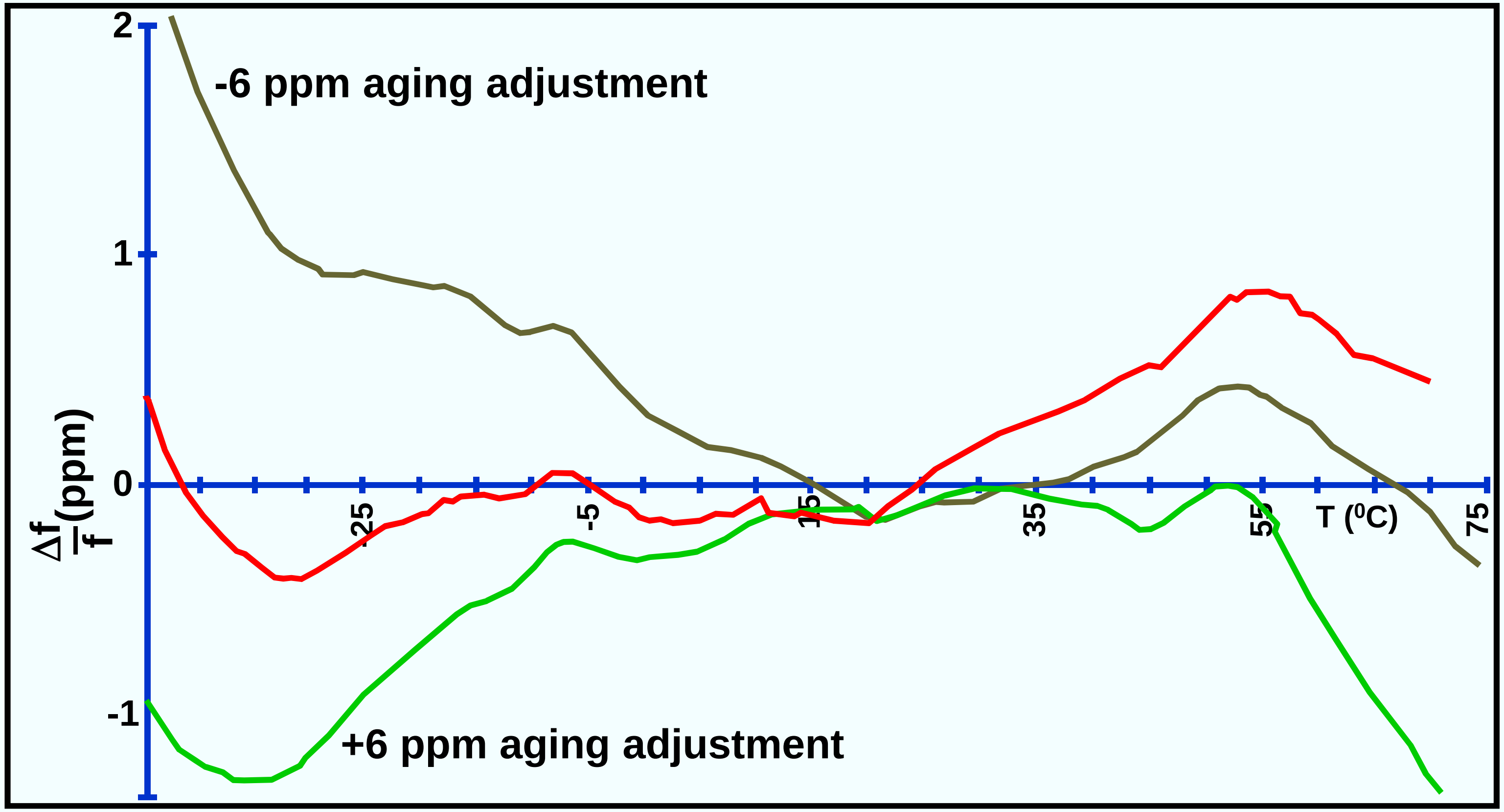


OCXO Retrace



In (a), the oscillator was kept on continuously while the oven was cycled off and on. In (b), the oven was kept on continuously while the oscillator was cycled off and on.

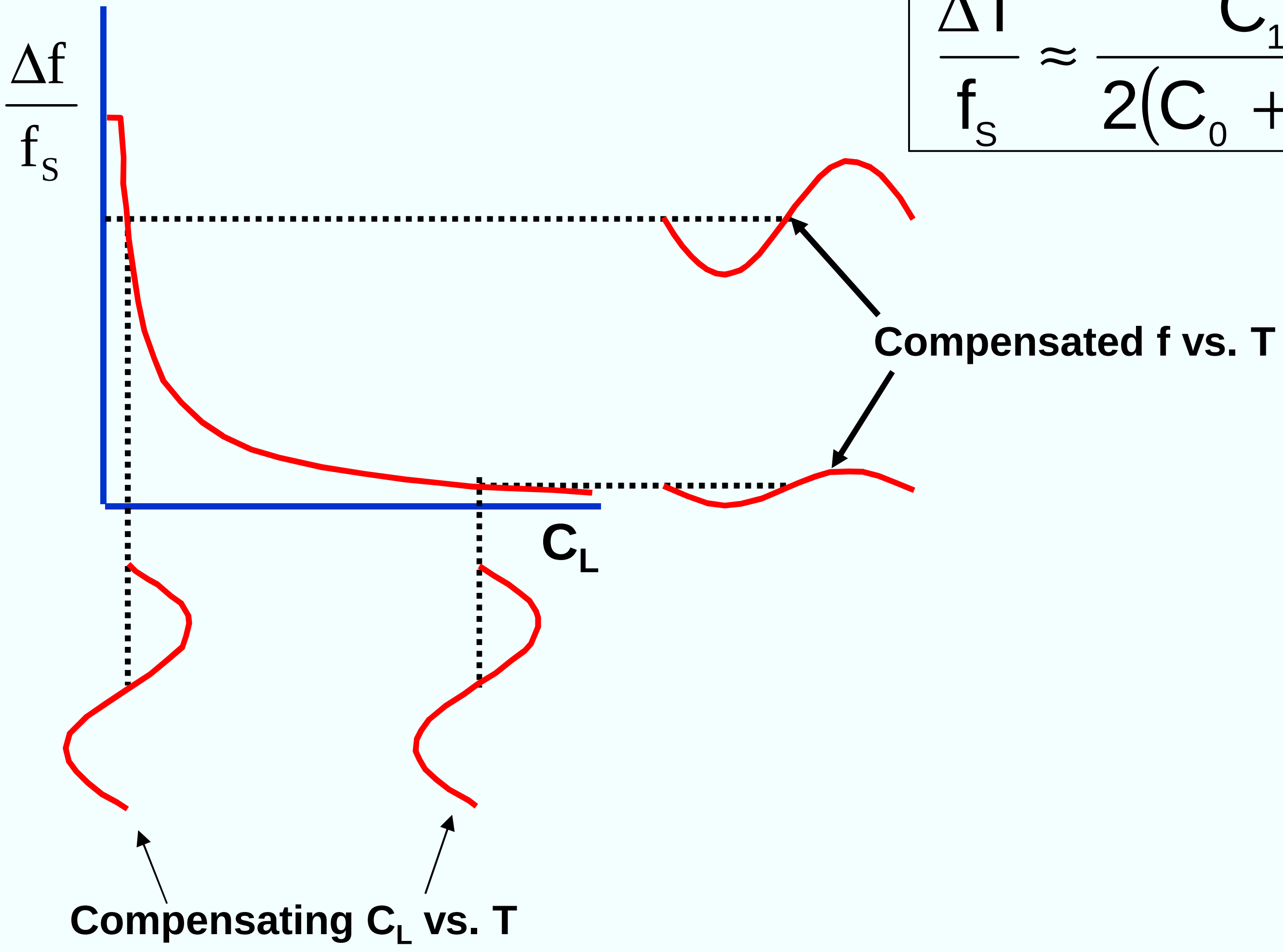
TCXO Trim Effect



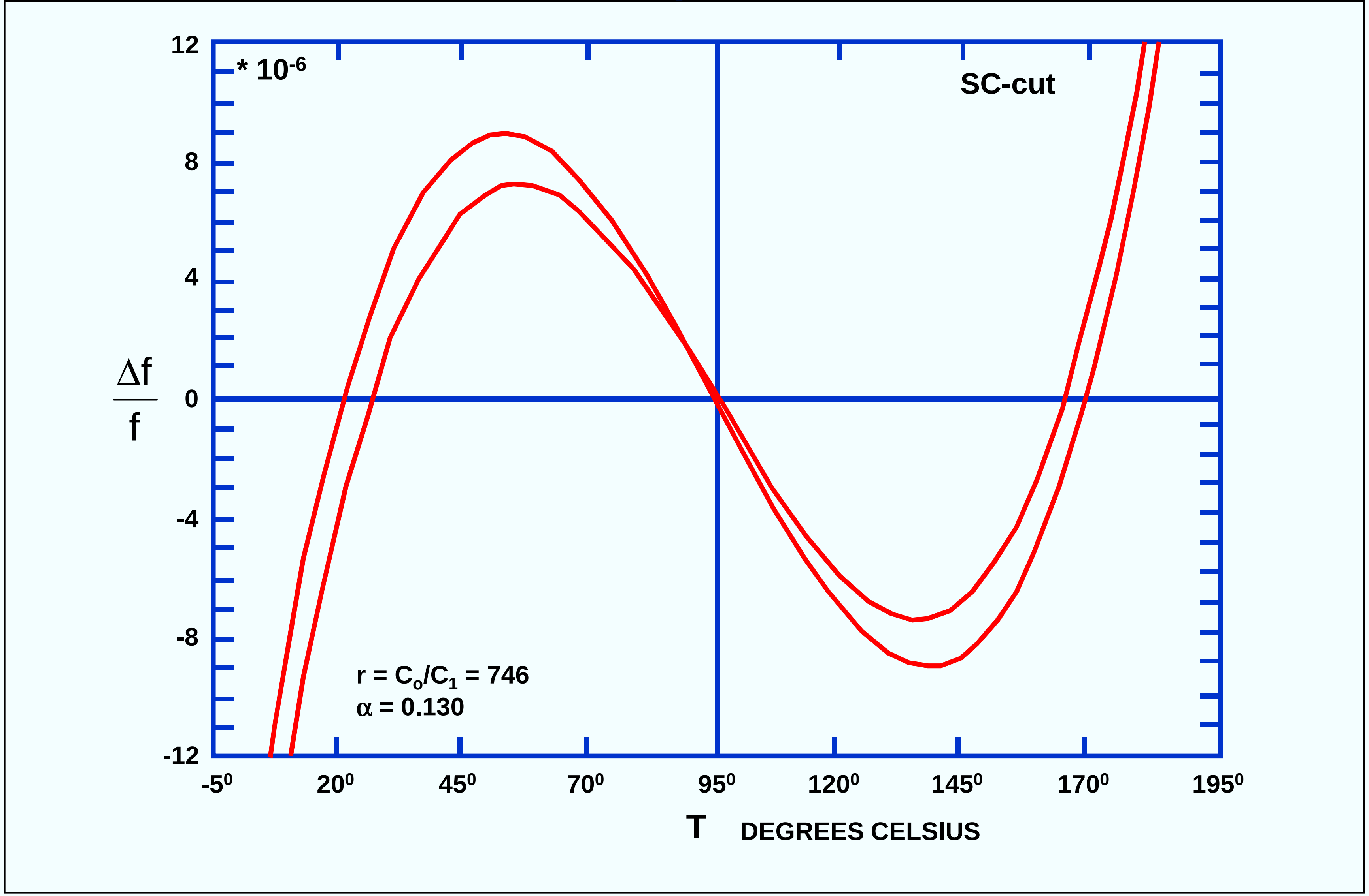
In TCXO's, temperature sensitive reactances are used to compensate for f vs. T variations. A variable reactance is also used to compensate for TCXO aging. The effect of the adjustment for aging on f vs. T stability is the "trim effect". Curves show f vs. T stability of a "0.5 ppm TCXO," at zero trim and at ±6 ppm trim. (Curves have been vertically displaced for clarity.)

Why the Trim Effect?

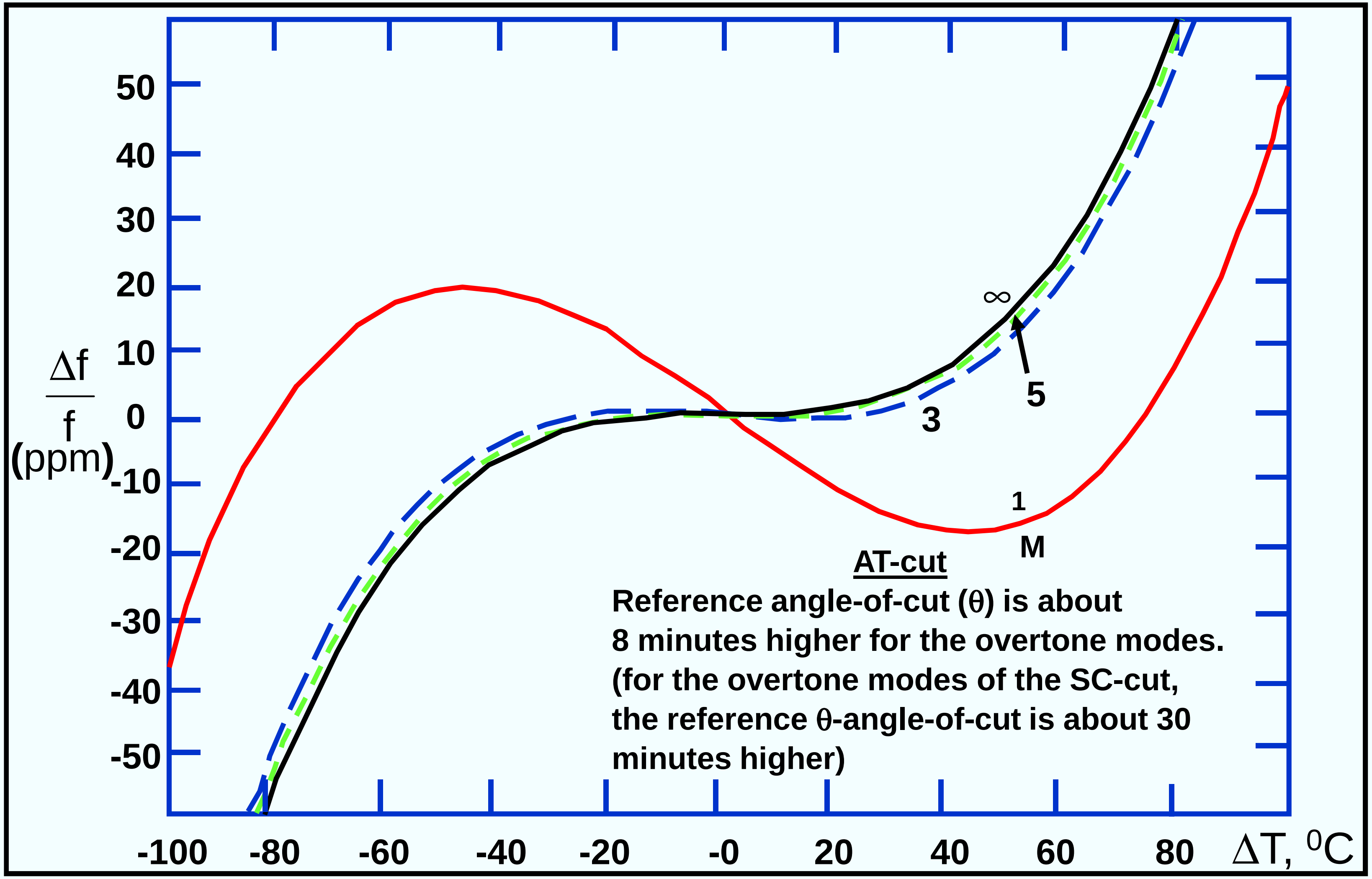
$$\frac{\Delta f}{f_S} \approx \frac{C_1}{2(C_0 + C_L)}$$



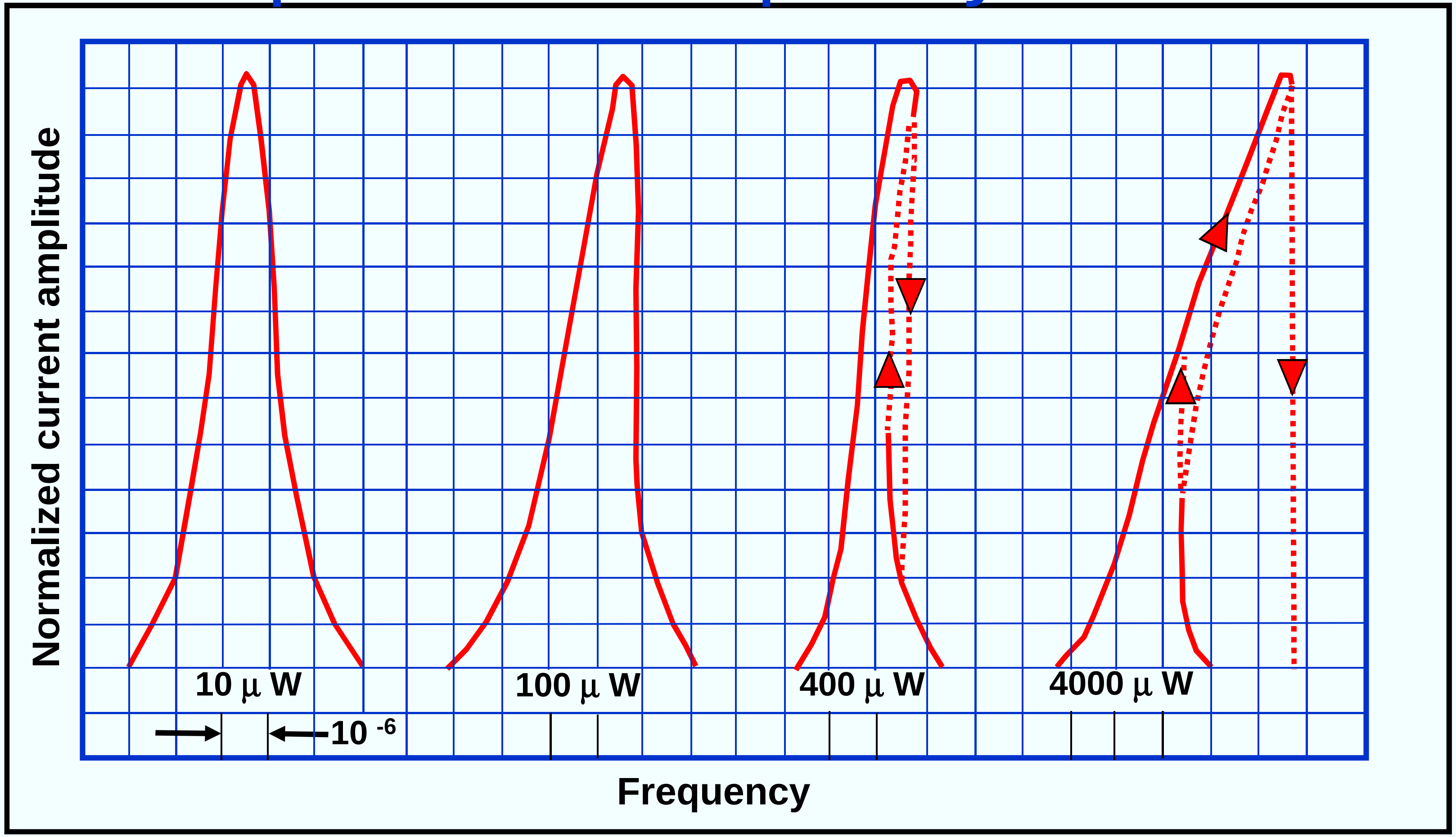
Effects of Load Capacitance on f vs. T



Effects of Harmonics on f vs. T

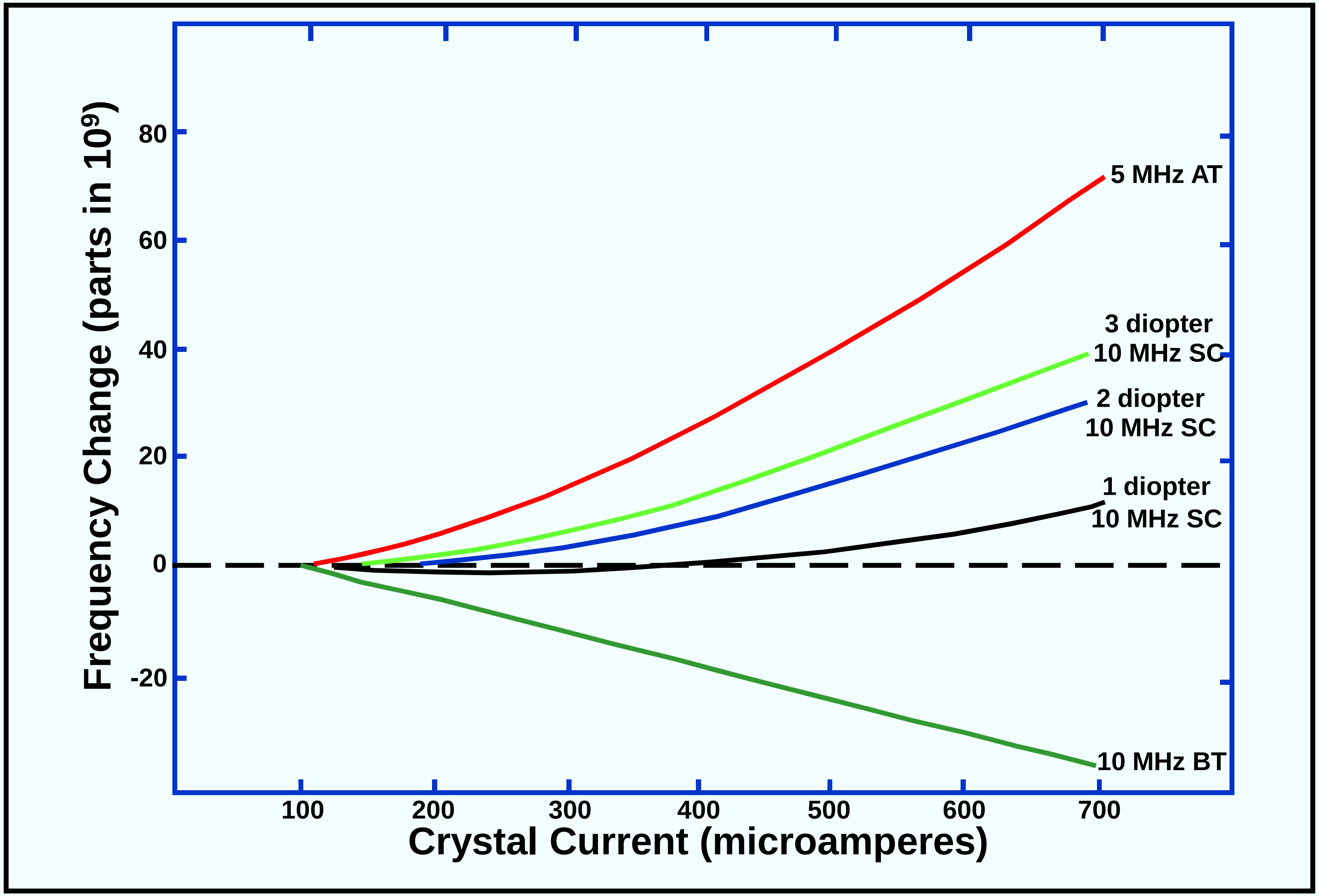


Amplitude - Frequency Effect

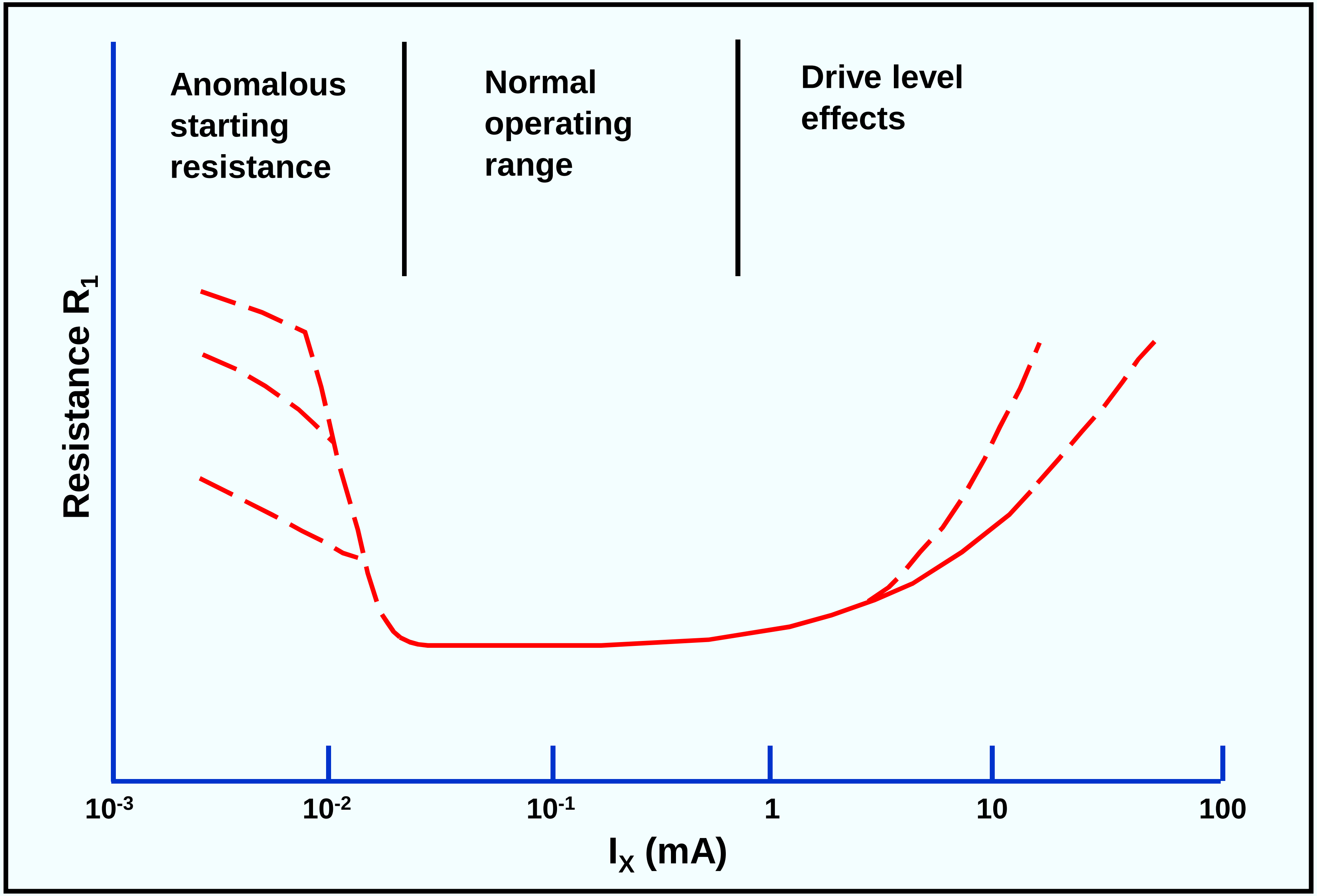


At high drive levels, resonance curves become asymmetric due to the nonlinearities of quartz.

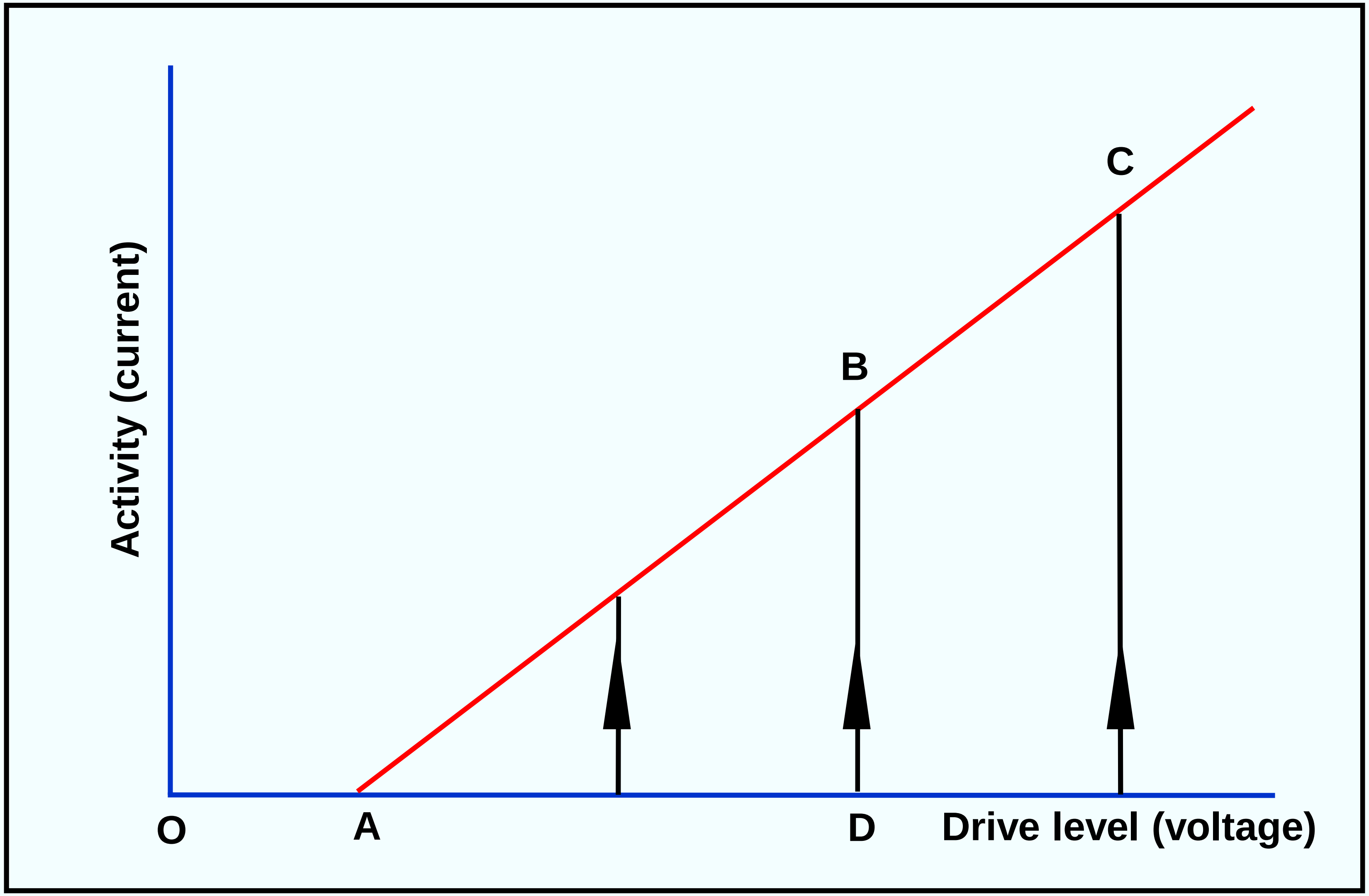
Frequency vs. Drive Level



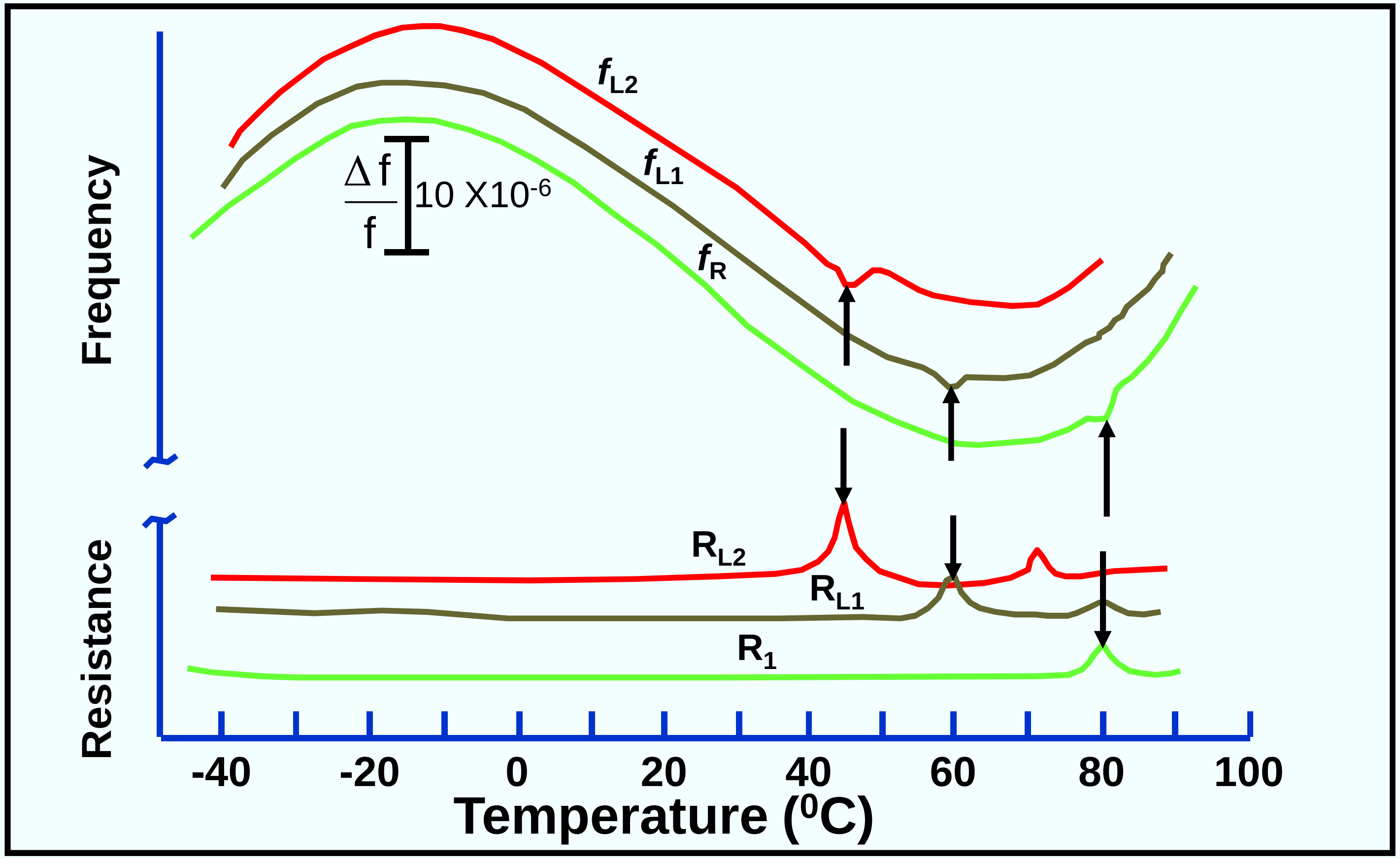
Drive Level vs. Resistance



Second Level of Drive Effect

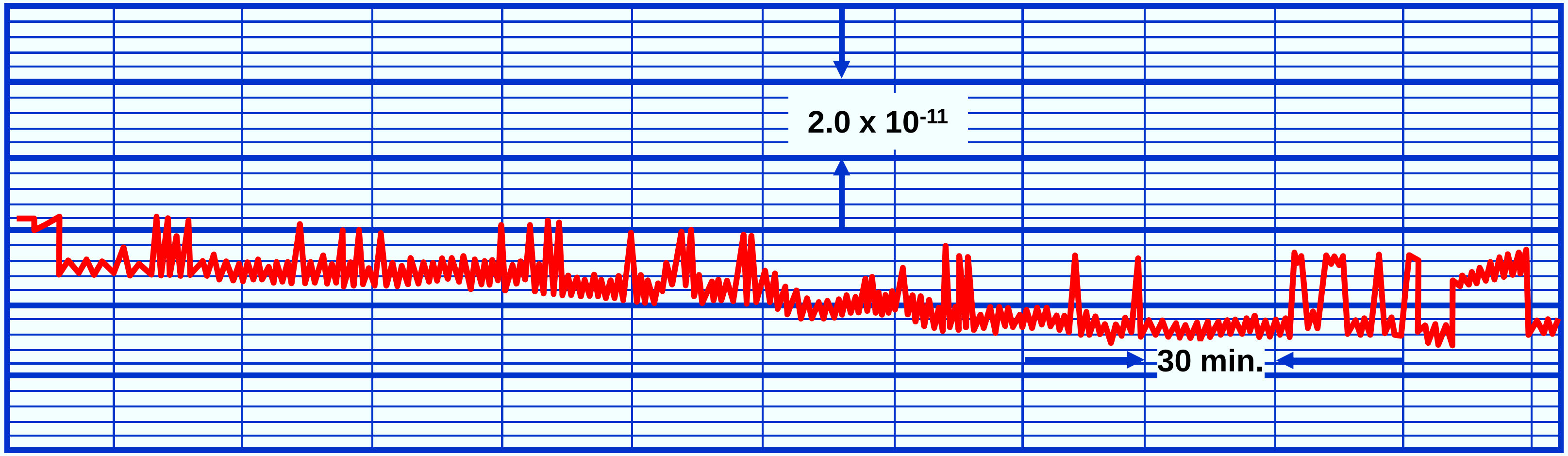


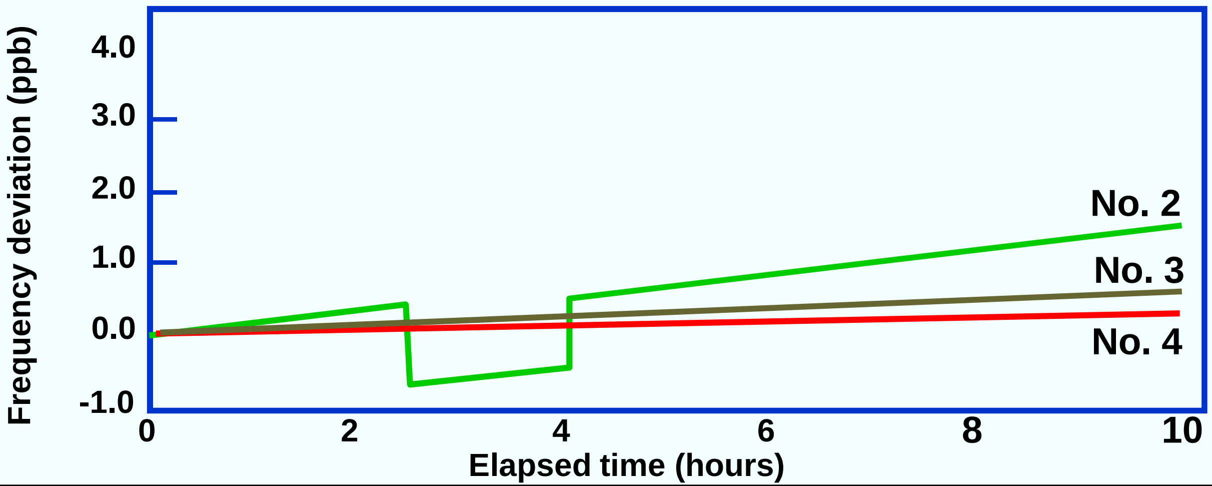
Activity Dips



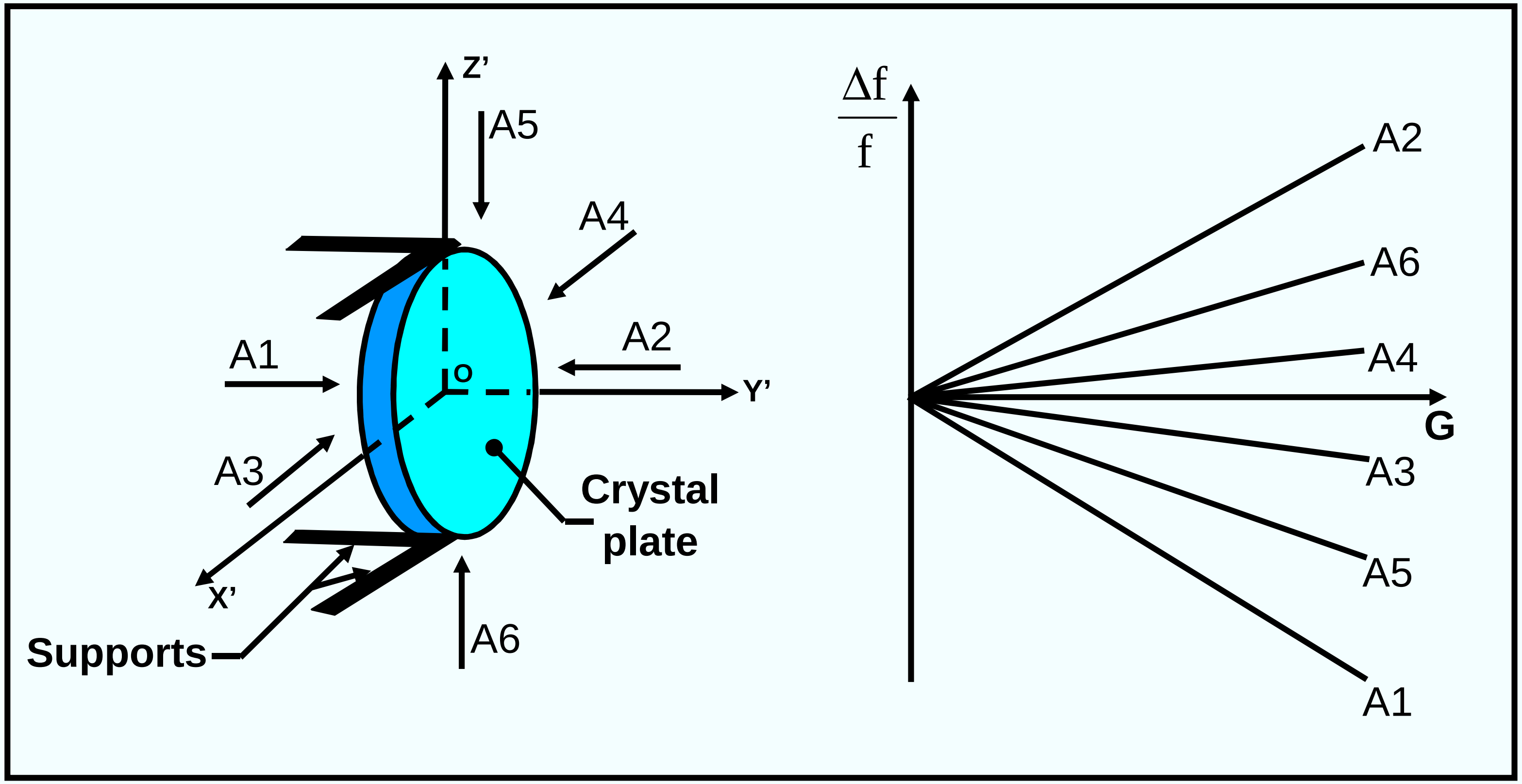
Activity dips in the f vs. T and R vs. T when operated with and without load capacitors. Dip temperatures are a function of C_L, which indicates that the dip is caused by a mode (probably flexure) with a large negative temperature coefficient.

Frequency Jumps





Acceleration vs. Frequency Change



Frequency shift is a function of the magnitude and direction of the acceleration, and is usually linear with magnitude up to at least 50 g's.

Acceleration Is Everywhere

Environment	**Acceleration** typical levels*, in g's	**Δf/f** $x10^{-11}$, for $1x10^{-9}$/g oscillator
Buildings**, quiesent	0.02 rms	2
Tractor-trailer (3-80 Hz)	0.2 peak	20
Armored personnel carrier	0.5 to 3 rms	50 to 300
Ship - calm seas	0.02 to 0.1 peak	2 to 10
Ship - rough seas	0.8 peak	80
Propeller aircraft	0.3 to 5 rms	30 to 500
Helicopter	0.1 to 7 rms	10 to 700
Jet aircraft	0.02 to 2 rms	2 to 200
Missile - boost phase	15 peak	1,500
Railroads	0.1 to 1 peak	10 to 100
Spacecraft	Up to 0.2 peak	Up to 20

* Levels at the oscillator depend on how and where the oscillator is mounted
Platform resonances can greatly amplify the acceleration levels.

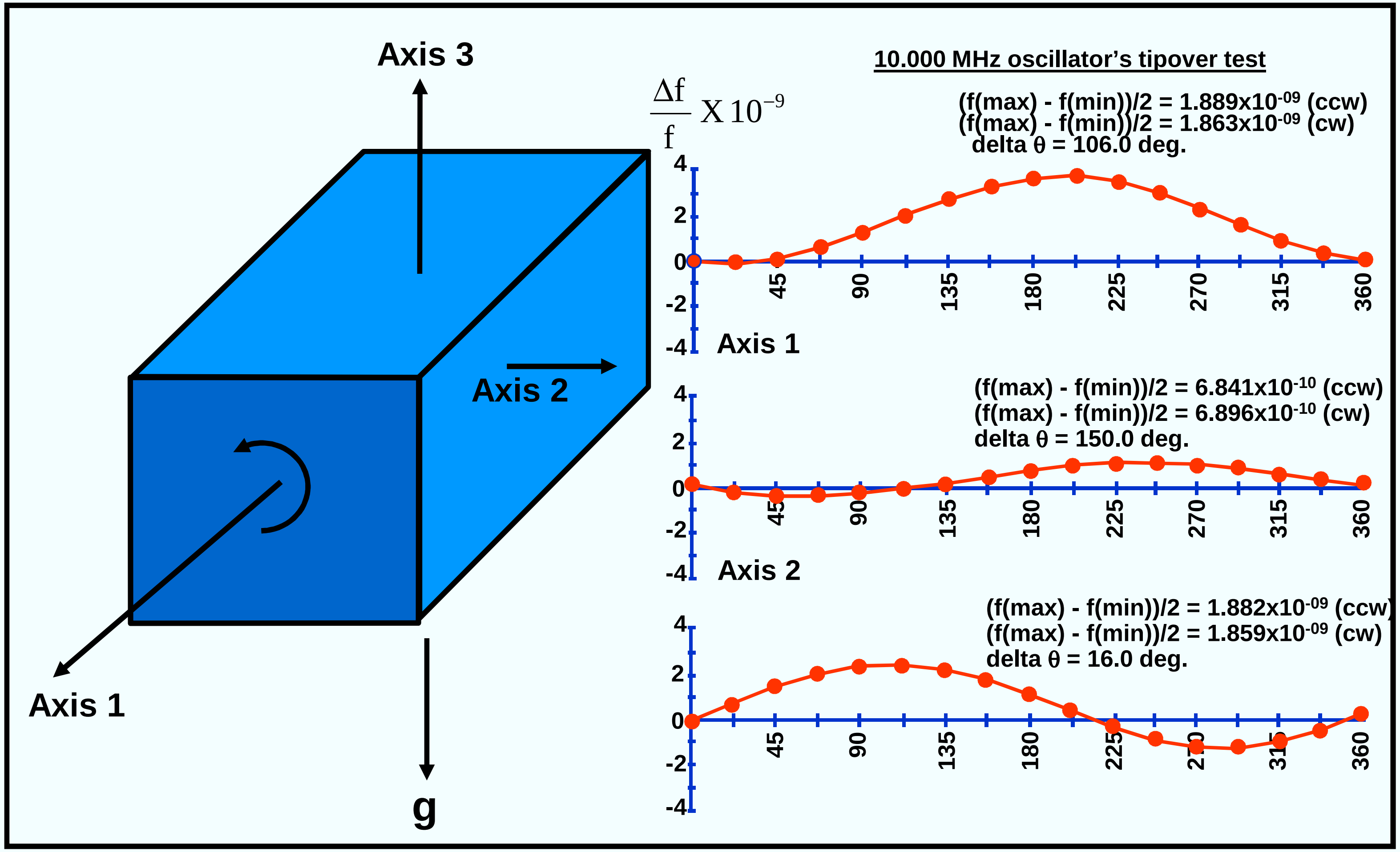
** Building vibrations can have significant effects on noise measurements

Acceleration Affects "Everything"

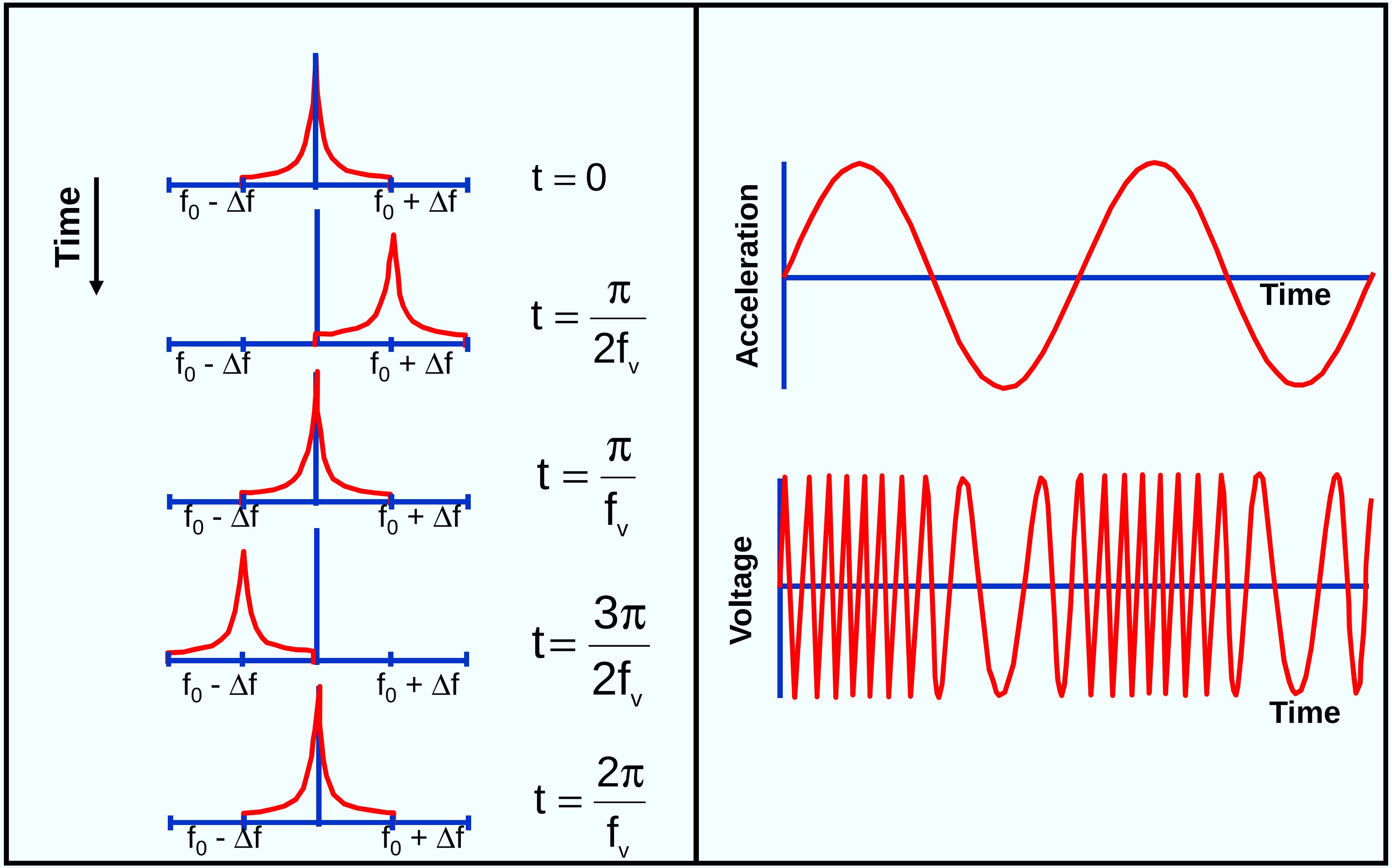
- Acceleration → Force → Deformation (strain) →
  Change in material and device properties - to some level

- Examples:
  - Quartz resonator frequency
  - Amplifier gain (strain changes semiconductor band structure)
  - Laser diode emission frequencies
  - Optical properties - fiber index of refraction (acoustooptics)
  - Cavity frequencies
  - DRO frequency (strain changes dielectric constants)
  - Atomic clock frequencies
  - Stray reactances
  - Clock rates (relativistic effects)

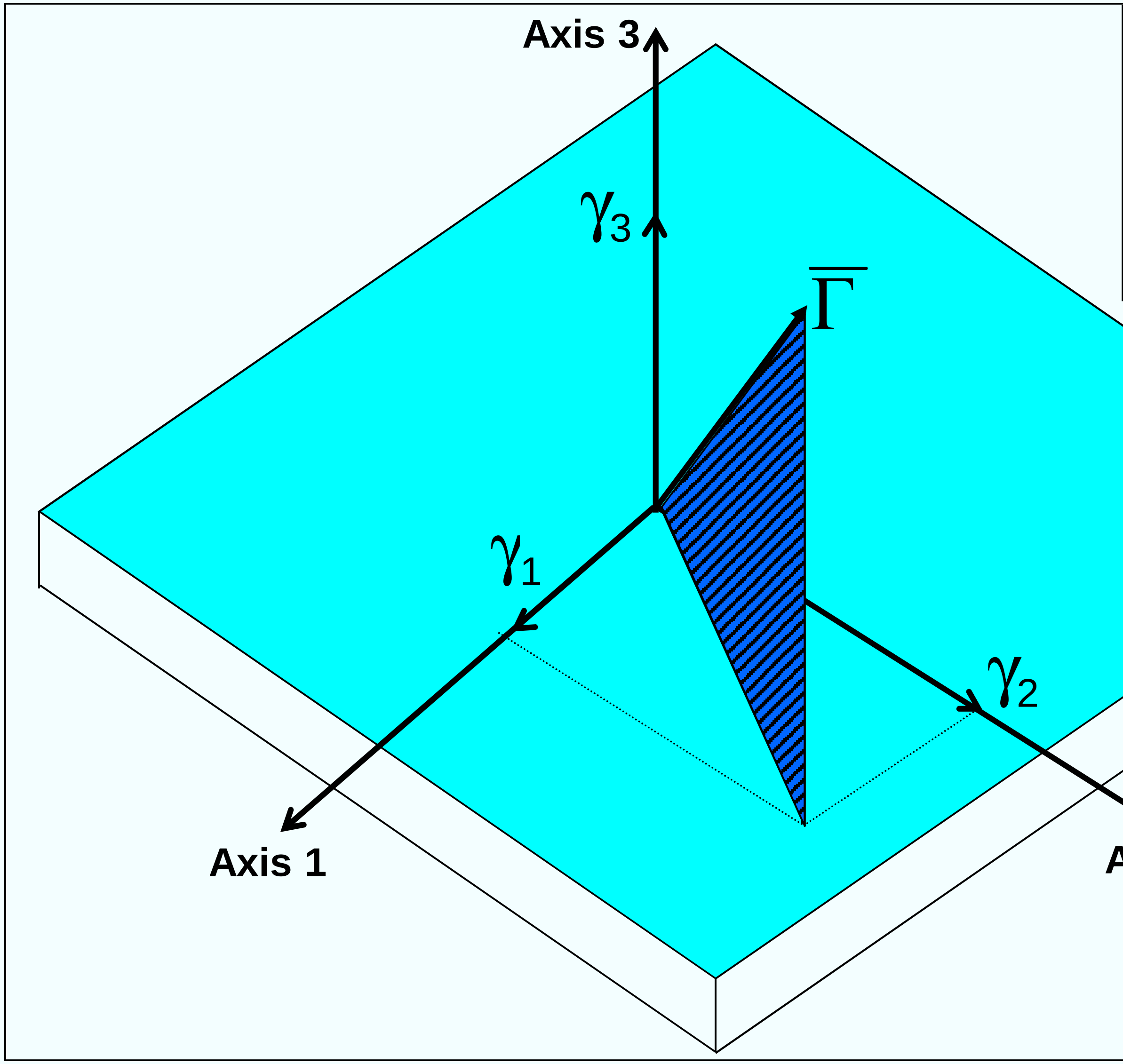
2-g Tipover Test
(Δf vs. attitude about three axes)



Sinusoidal Vibration Modulated Frequency



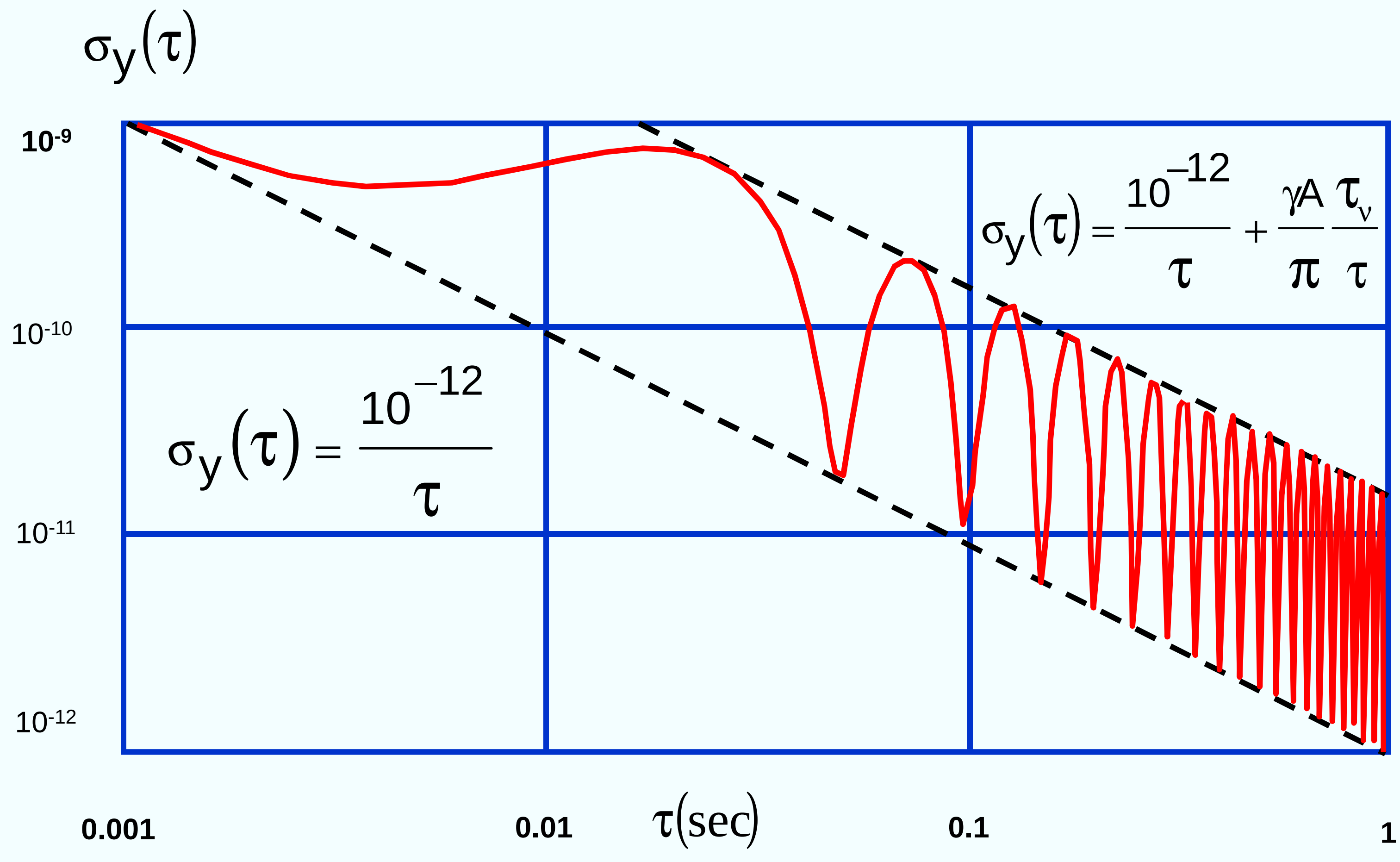
Acceleration Sensitivity Vector



$$\overline{\Gamma} = \gamma_1 \hat{i} + \gamma_2 \hat{j} + \gamma_3 \hat{k}$$

$$\Gamma = \sqrt{\gamma_1^2 + \gamma_2^2 + \gamma_3^2}$$

Vibration-Induced Allan Deviation Degradation



Example shown: f_v = 20, Hz A = 1.0 g along Γ, ‖Γ‖ = 1 x 10^{-9}/g

Vibration-Induced Phase Excursion

The phase of a vibration modulated signal is

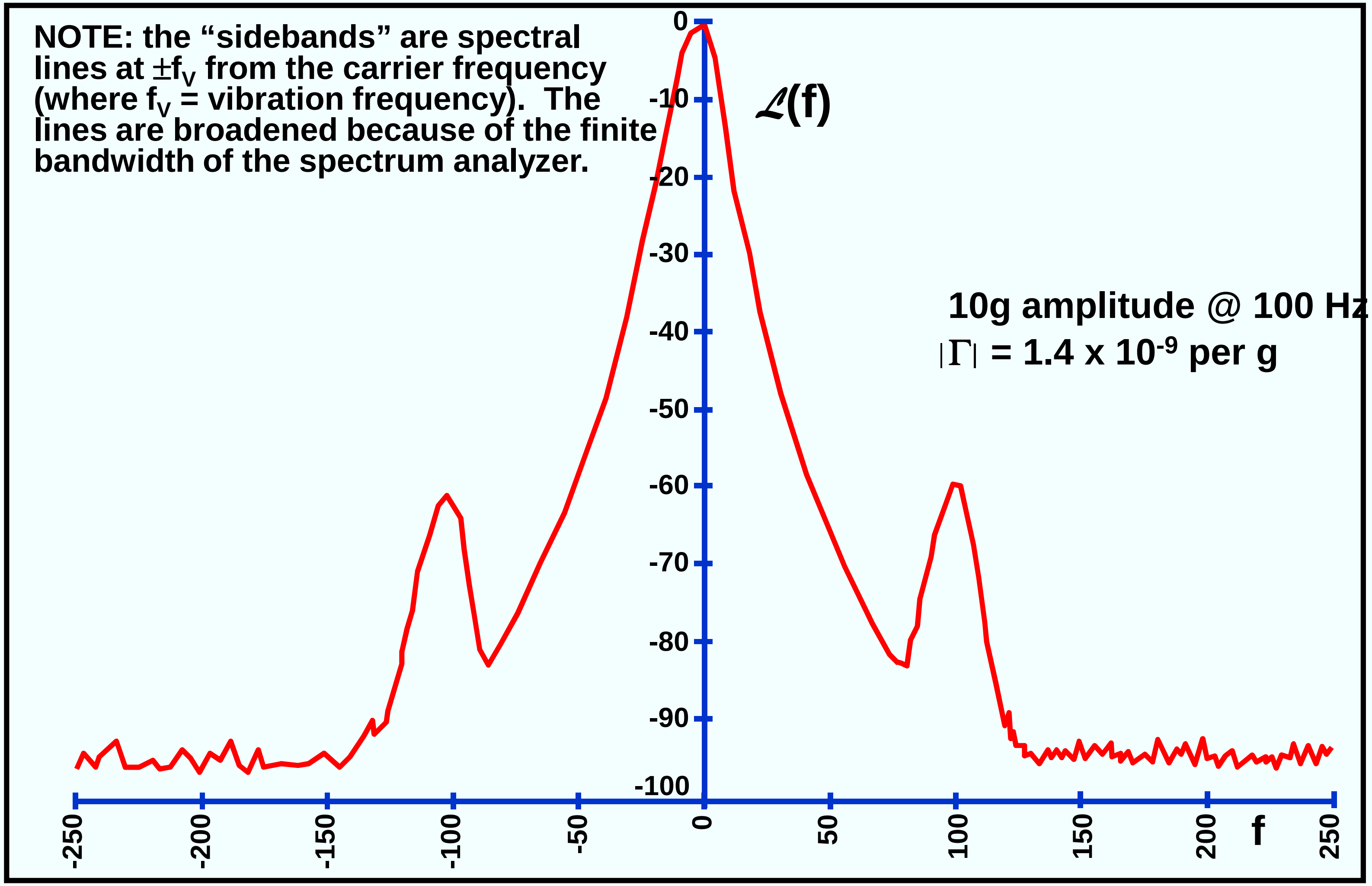
$$\varphi(t) = 2\pi f_0 t + \left(\frac{\Delta f}{f_v}\right)\sin(2\pi f_v t)$$

When the oscillator is subjected to a sinusoidal vibration, the peak phase excursion is

$$\Delta\varphi_{peak} = \frac{\Delta f}{f_v} = \frac{(\overline{\Gamma} \bullet \overline{A})f_0}{f_v}$$

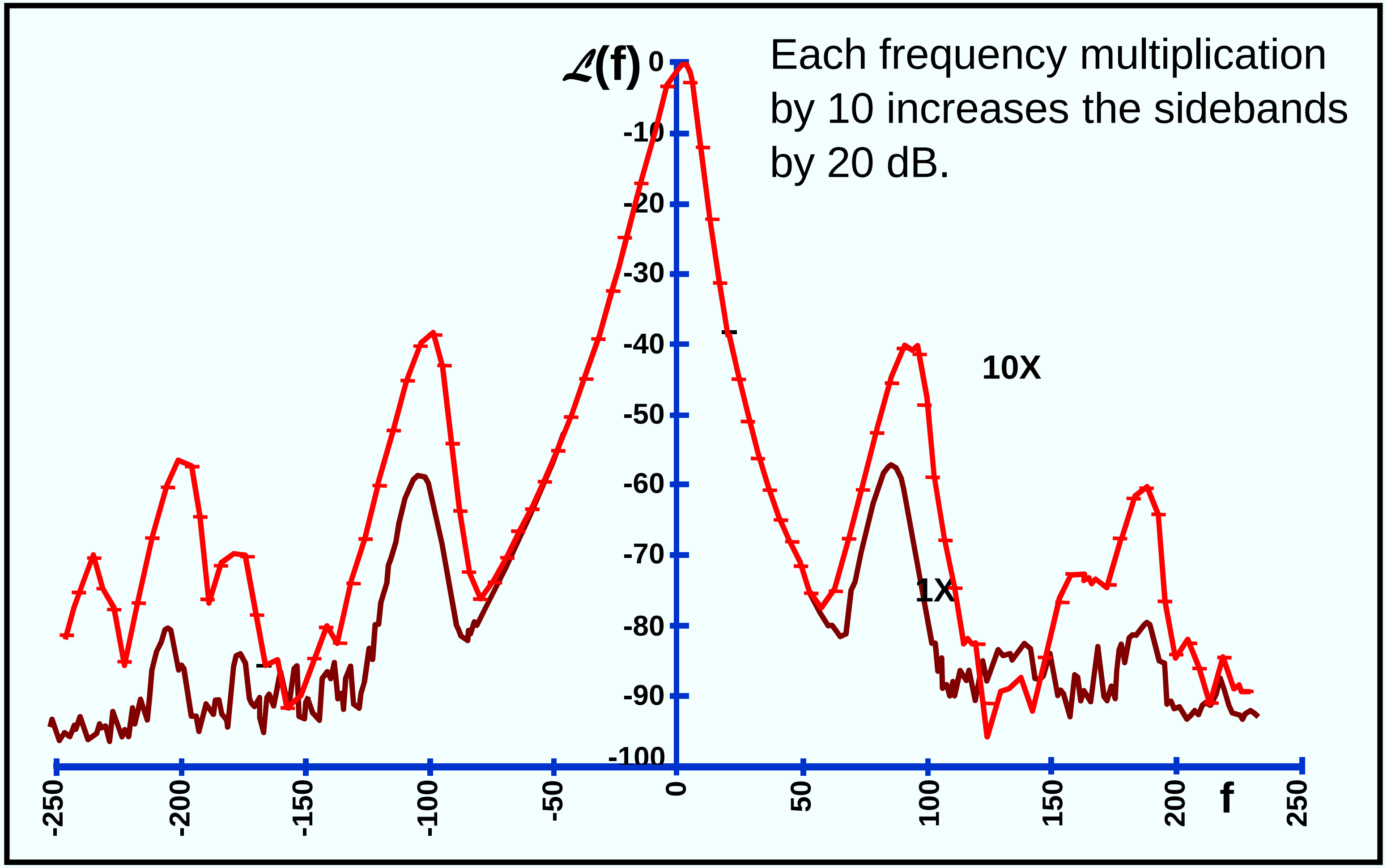
<u>Example</u>: if a 10 MHz, 1 x 10^{-9}/g oscillator is subjected to a 10 Hz sinusoidal vibration of amplitude 1g, the peak vibration-induced phase excursion is 1 x 10^{-3} radian. If this oscillator is used as the reference oscillator in a 10 GHz radar system, the peak phase excursion at 10GHz will be 1 radian. Such a large phase excursion can be catastrophic to the performance of many systems, such as those which employ phase locked loops (PLL) or phase shift keying (PSK).

Vibration-Induced Sidebands



Vibration-Induced Sidebands

After Frequency Multiplication



Sine Vibration-Induced Phase Noise

Sinusoidal vibration produces spectral lines at $\pm f_v$ from the carrier, where f_v is the vibration frequency.

$$\mathcal{L}'(f_v) = 20 \log \left(\frac{\bar{\Gamma} \bullet \bar{A} f_0}{2 f_v} \right)$$

e.g., if $|\bar{\Gamma}|$= 1 x 10^{-9}/g and f_0 = 10 MHz, then even if the oscillator is completely **noise free at rest**, the phase “noise” i.e., the spectral lines, due solely to a sine vibration level of 1g will be;

Vibr. freq., f_v, in Hz	$\mathcal{L}'(f_v)$, in dBc
1	-46
10	-66
100	-86
1,000	-106
10,000	-126

Random Vibration-Induced Phase Noise

Random vibration's contribution to phase noise is given by:

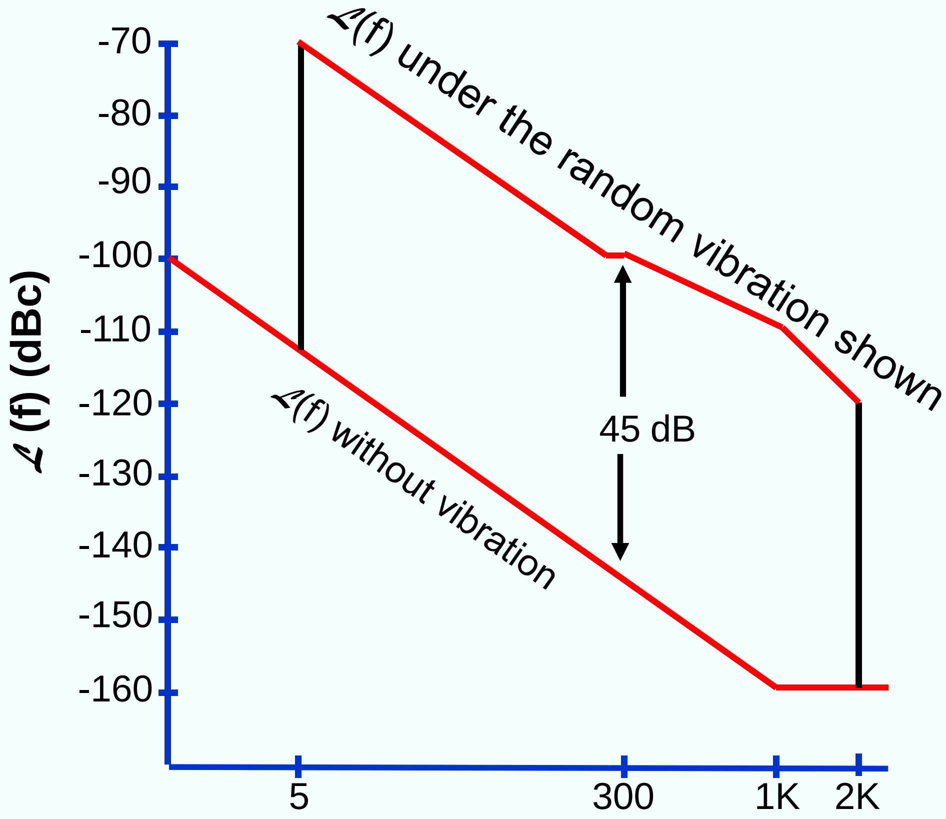
$$\mathcal{L}(f) = 20 \log\left(\frac{\overline{\Gamma} \bullet \overline{A} f_0}{2f}\right), \quad \text{where } |\overline{A}| = [(2)(PSD)]^{1/2}$$

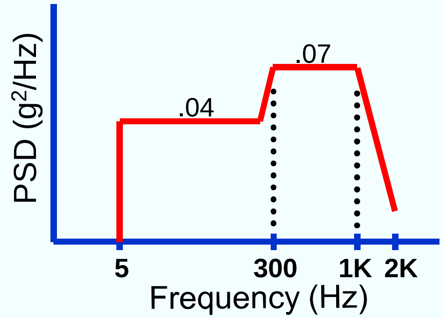
e.g., if $|\overline{\Gamma}|$= 1 x 10^{-9}/g and f_0 = 10 MHz, then even if the oscillator is completely **noise free at rest**, the phase "noise" i.e., the spectral lines, due solely to a vibration of power spectral density, PSD = 0.1 g^2/Hz will be:

Offset freq., f, in Hz	$\mathcal{L}$'(f), in dBc/Hz
1	-53
10	-73
100	-93
1,000	-113
10,000	-133

Random-Vibration-Induced Phase Noise

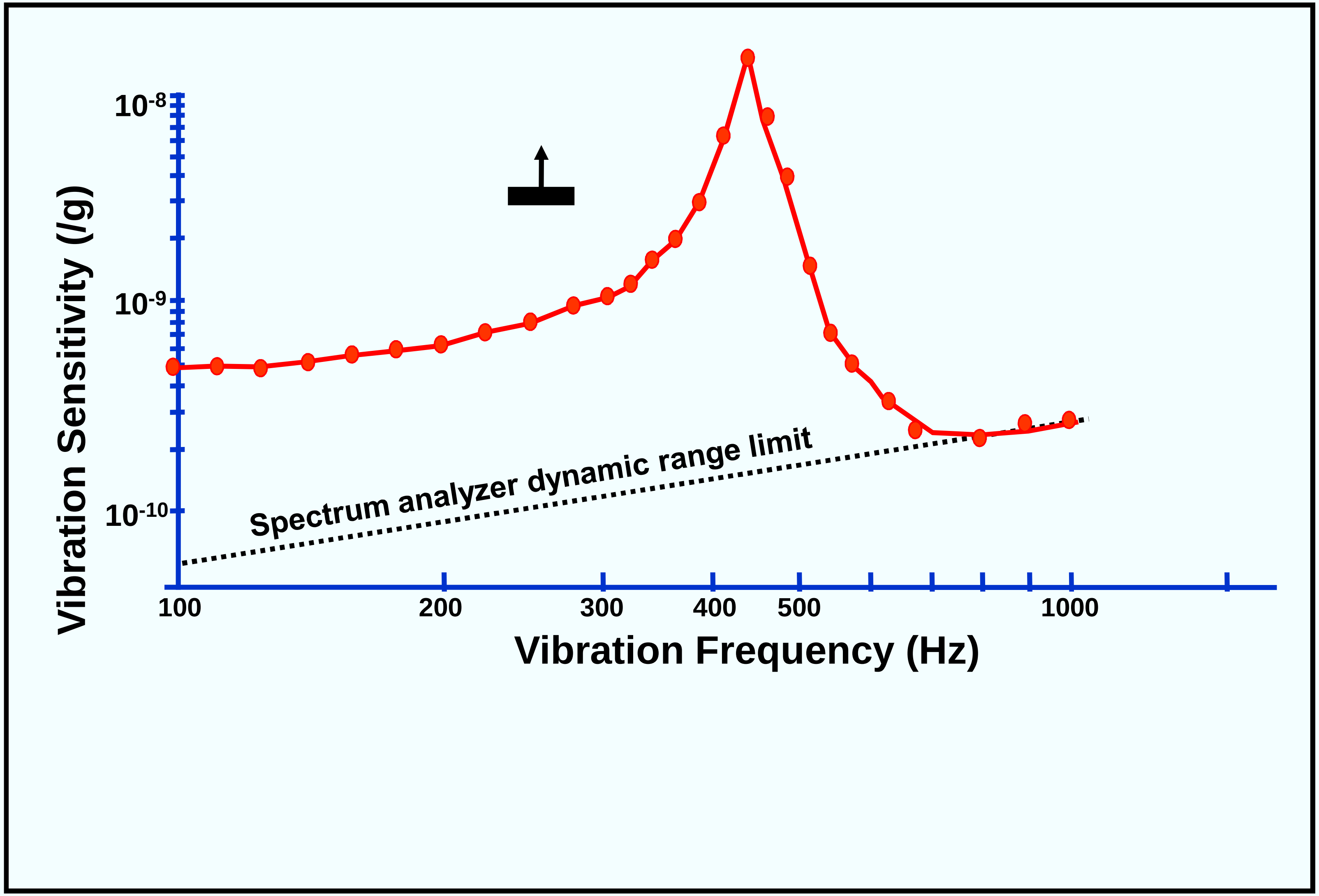
Phase noise under vibration is for $\Gamma = 1 \times 10^{-9}$ per g and f = 10 MHz





Typical aircraft random vibration envelope

Acceleration Sensitivity vs. Vibration Frequency

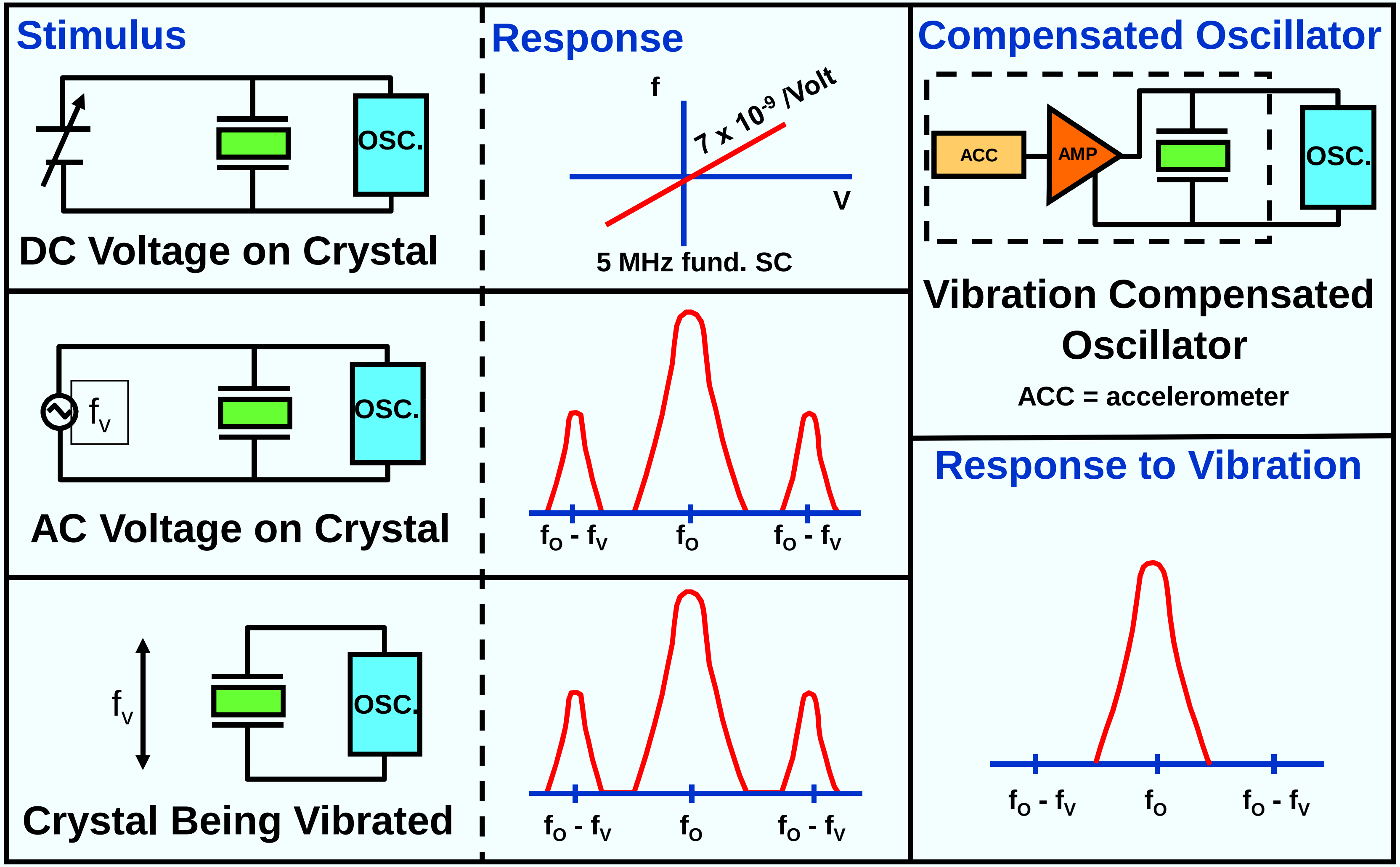


Acceleration Sensitivity of Quartz Resonators

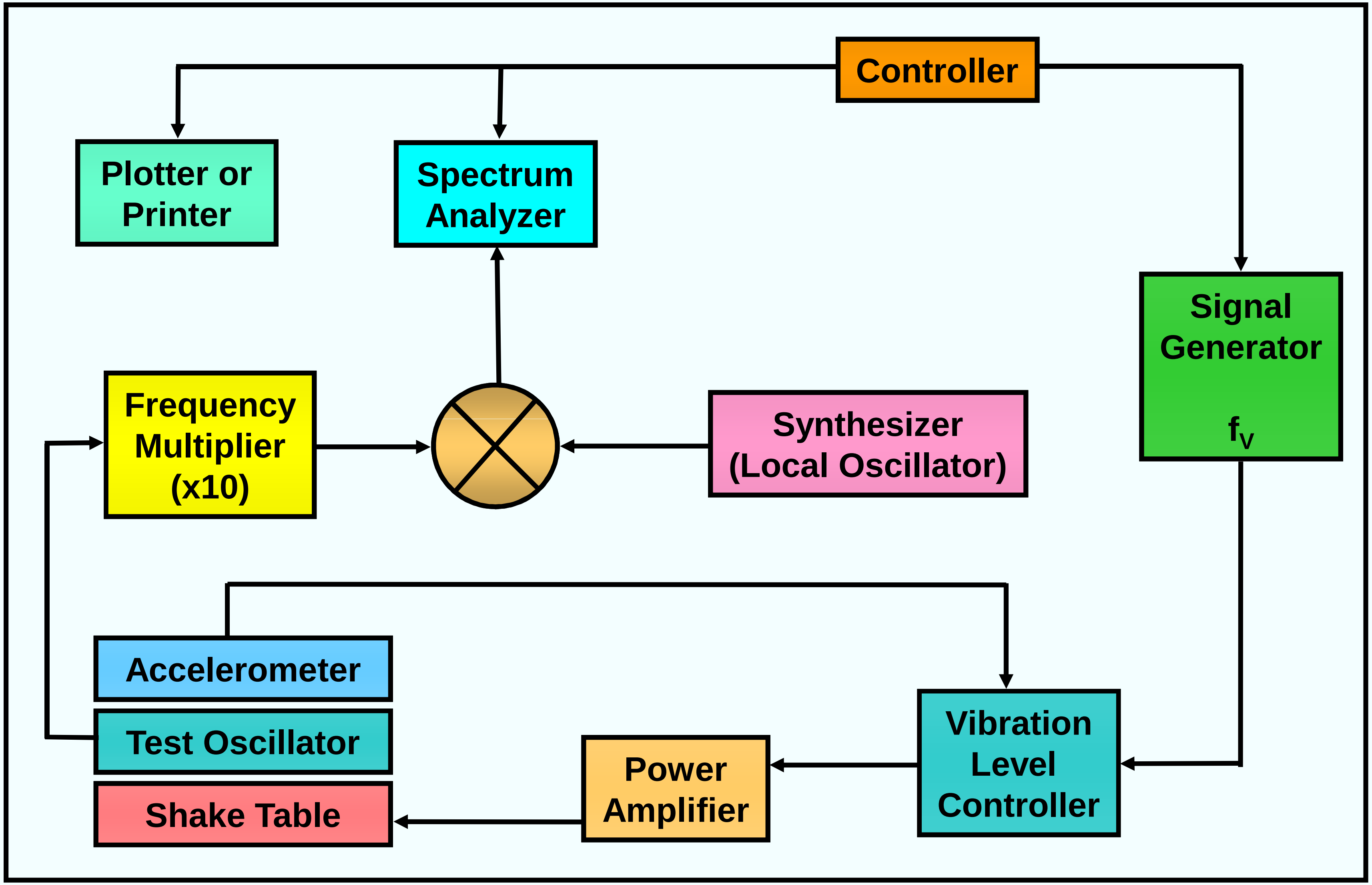
Resonator acceleration sensitivities range from the low parts in 10^{10} per g for the best commercially available SC-cuts, to parts in 10^{7} per g for tuning-fork-type watch crystals. When a wide range of resonators were examined: AT, BT, FC, IT, SC, AK, and GT-cuts; 5 MHz 5th overtones to 500 MHz fundamental mode inverted mesa resonators; resonators made of natural quartz, cultured quartz, and swept cultured quartz; numerous geometries and mounting configurations (including rectangular AT-cuts); nearly all of the results were within a factor of three of $1x10^{-9}$ per g. On the other hand, the fact that a few resonators have been found to have sensitivities of less than $1x10^{-10}$ per g indicates that the observed acceleration sensitivities are not due to any inherent natural limitations.

Theoretical and experimental evidence indicates that the major variables yet to be controlled properly are the mode shape and location (i.e., the amplitude of vibration distribution), and the strain distribution associated with the mode of vibration. Theoretically, when the mounting is completely symmetrical with respect to the mode shape, the acceleration sensitivity can be zero, but tiny changes from this ideal condition can cause a significant sensitivity. Until the acceleration sensitivity problem is solved, acceleration compensation and vibration isolation can provide lower than $1x10^{-10}$ per g, for a limited range of vibration frequencies, and at a cost.

Vibration Compensation

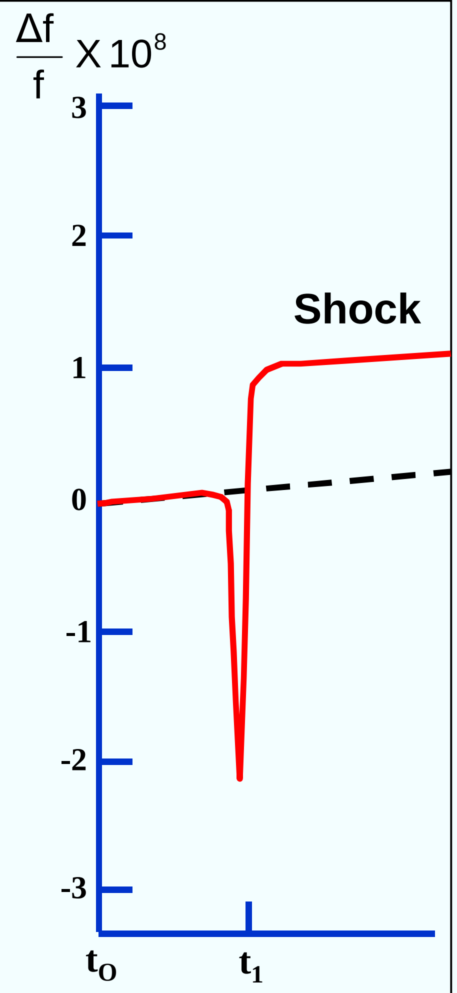


Vibration Sensitivity Measurement System



Shock

The frequency excursion during a shock is due to the resonator's stress sensitivity. The magnitude of the excursion is a function of resonator design, and of the shock induced stresses on the resonator (resonances in the mounting structure will amplify the stresses.) The permanent frequency offset can be due to: shock induced stress changes, a change in (particulate) contamination on the resonator surfaces, and changes in the oscillator circuitry. Survival under shock is primarily a function of resonator surface imperfections. Chemical-polishing-produced scratch-free resonators have survived shocks up to 36,000 g in air gun tests, and have survived the shocks due to being fired from a 155 mm howitzer (16,000 g, 12 ms duration).



Other Effects on Stability

- **Electric field -** affects doubly-rotated resonators; e.g., a voltage on the electrodes of a 5 MHz fundamental mode SC-cut resonator results in a $\Delta f/f = 7 \times 10^{-9}$ per volt. The voltage can also cause sweeping, which can affect the frequency (of all cuts), even at normal operating temperatures.

- **Magnetic field -** quartz is diamagnetic, however, magnetic fields can induce Eddy currents, and will affect magnetic materials in the resonator package and the oscillator circuitry. Induced ac voltages can affect varactors, AGC circuits and power supplies. Typical frequency change of a "good" quartz oscillator is $<<10^{-10}$ per gauss.

- **Ambient pressure (altitude) -** deformation of resonator and oscillator packages, and change in heat transfer conditions affect the frequency.

- **Humidity -** can affect the oscillator circuitry, and the oscillator's thermal properties, e.g., moisture absorbed by organics can affect dielectric constants.

- **Power supply voltage, and load impedance -** affect the oscillator circuitry, and indirectly, the resonator's drive level and load reactance. A change in load impedance changes the amplitude or phase of the signal reflected into the oscillator loop, which changes the phase (and frequency) of the oscillation. The effects can be minimized by using a (low noise) voltage regulator and buffer amplifier.

- **Gas permeation -** stability can be affected by excessive levels of atmospheric hydrogen and helium diffusing into "hermetically sealed" metal and glass enclosures (e.g., hydrogen diffusion through nickel resonator enclosures, and helium diffusion through glass Rb standard bulbs).

Interactions Among Influences

In attempting to measure the effect of a single influence, one often encounters interfering influences, the presence of which may or may not be obvious.

Measurement	**Interfering Influence**
Resonator aging	ΔT due to oven T (i.e., thermistor) aging Δ drive level due to osc. circuit aging
Short term stability	Vibration
Vibration sensitivity	Induced voltages due to magnetic fields
2-g tipover sensitivity	ΔT due to convection inside oven
Resonator f vs. T (static)	Thermal transient effect, humidity T-coefficient of load reactances
Radiation sensitivity	ΔT, thermal transient effect, aging